J. DIFFERENTIAL GEOMETRY
13 (1978) 79-86

ANTI-HOLOMORPHIC AUTOMORPHISMS OF THE EXCEPTIONAL SYMMETRIC DOMAINS

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Introduction

A serious fault of the theory described in [8] and called "real forms of hermitian symmetric spaces" was the lack of information about the exceptional symmetric domains. This gap has been filled, and the new results are given here.

Let me now express my thanks to Professor Kuga for having posed the problems of [7], [8], and to Professors Borel, Helgason, and Langlands for several enlightening discussions about the present work. In particular, while the idea behind the "hard part" of Lemma (2.4) is my own, Borel is to be credited for adding the necessary rigor to my original argument. Various improvements in my original paper were suggested by the referee, especially the use of Theorem (2.10) in § 4.

1. The problem

Let X denote a hermitian symmetric space of noncompact type (for short, symmetric domain), $x_0 \in X$ a base point, $\mathscr{C}$ the set of anti-holomorphic involutive automorphisms of X, and $\mathscr{C}_0 = \{\sigma \in \mathscr{C} \mid \sigma(x_0) = x_0\}$. If G^h is the group of holomorphic automorphisms of X, and K^h the isotropy group at x_0, we have $X \approx G^h/K^h$. For any $\sigma \in \mathscr{C}$, we call $X^\sigma = \{x \in X \mid \sigma(x) = x\}$ the *real form* of X *associated to* σ. G^h acts by conjugation on $\mathscr{C}$, and K^h preserves $\mathscr{C}_0$. We call the quotient $\mathscr{C}/G^h$ the set of *complex conjugations* of X. Representing a conjugation by a $\sigma \in \mathscr{C}$ (or even $\mathscr{C}_0$ by Remark 2.3) we associate a real form to each conjugation. Another representative σ' for the same conjugation is G^h-conjugate to σ, hence the real form associated to a conjugation is well determined, up to isometry.

In Theorems (4.3) and (4.4) we give $\mathscr{C}/G^h$ and the associated real forms for the two exceptional symmetric domains. The theorem in [8] on the conjugations of a symmetric domain without exceptional factor now applies with no restriction. It follows that, in general, distinct conjugations have nonisometric real forms; we know no *a priori* reason for this.

The next section (§ 2) applies more generally than just to the exceptional sym-

Communicated by S. Sternberg, November 5, 1975. Supported by National Science Foundation grant MPS 72-05055 A02.