J. DIFFERENTIAL GEOMETRY
13 (1978) 43-50

COMPACT REAL HYPERSURFACES WITH CONSTANT MEAN CURVATURE OF A COMPLEX PROJECTIVE SPACE

MASAFUMI OKUMURA

Introduction

The differential geometry of hypersurfaces of a Riemannian manifold of constant curvature and complex hypersurfaces of a Kaehlerian manifold has been studied for a long time. In particular, many global results have been obtained (for example [1], [3]) since the establishment of J. Simons' formula [6] for the Laplacian of the second fundamental form. However, the differential geometry of real hypersurfaces of a Kaehlerian manifold has not been explored to any great extent, even in the case where the ambient manifold is a complex projective space CP^m. One of the main reasons for us not to be able to get many results on a real hypersurface is the lack of enough "words" to describe differential geometric properties of the hypersurface. For instance, totally geodesic hypersurfaces and totally umbilical hypersurfaces characterize respectively hyperplanes and hyperspheres, when the ambient manifold is a Euclidean space, and they respectively characterize great and small spheres, when the ambient manifold is a sphere. But if the ambient manifold is a CP^m, as a consequence of Codazzi equation, we know that there exist neither totally geodesic hypersurfaces nor totally umbilical hypersurfaces (for example, [7]). One way to overcome such poverty of vocabulary has been established by H. B. Lawson [2] who introduced the notion of generalized equator $M^c_{p,q}$ of a CP^m. His idea is to construct a circle bundle over a real hypersurface, which is compatible with the Hopf fibration. Thus we can use many words to characterize remarkable classes of submanifolds of a sphere. By making use of the second fundamental form and the fundamental tensor of submersion, the present author [4] gave a condition for the circle bundle over a real hypersurface of a CP^m to be a product of two spheres.

Keeping this point of view, in this paper we study compact real hypersurfaces of a CP^m with constant mean curvature.

In § 1, we review necessary results obtained in [2] and [4] for the use in § 3. In § 2 we compute the Laplacian of the length of the second fundamental form of a real hypersurface of a CP^m.

Communicated by I. M. Singer, November 19, 1975.