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PSEUDO-HERMITIAN STRUCTURES ON A REAL HYPERSURFACE

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Introduction

The invariance properties of a real hypersurface M (of real dimension $2n + 1$) in complex $(n + 1)$ space C^{n+1} with respectt o the infinite pseudo-group of biholomorphic transformations are the object of study in pseudo-conformal geometry. The systematic study of such properties for hypersurfaces with nondegenerate Levi form was first made by Cartan [2] in 1932. More recently, the study of invariants for such M was taken up by S. S. Chern and J. Moser [6]. A main aspect of the theory is the existence of a complete system of local differential invariants.

In this paper we take a somewhat different point of view. Such a manifold M has an integrable, nondegenerate, Cauchy-Riemann structure. In particular, there is a subbundle $H(M)$ of the tangent bundle $T(M)$ each fiber of which has the structure of a complex n-dimensional vector space. We single out a real nonvanishing one-form θ annihilating $H(M)$ and consider invariants of the pair (M, θ). (M, θ) will be called a pseudo-hermitian manifold.

In § 1 we apply the Cartan method of equivalence [3] to find a compete system of invariants. This results in a connection and curvature forms on the coframe bundle of M. These are not, in general, pseudo-conformal invariants; they depend on the choice of θ. In § 3 we consider the relation between these two systems of invariants. (3.8) gives a formula for the fourth order curvature tensor of Chern and Moser. A similar formula was given by Bochner [1] as a formal analogue of the conformal curvature tensor for a Kähler manifold. Here a geometric interpretation of the formula is given. In § 4 we apply the theory to some examples. It is shown that an ellipsoid is not, in general, equivalent to a sphere.

Also, the author wishes to remark that the theory developed here provides a complete system of invariants for nondegenerate real hypersurfaces under volume-preserving biholomorphic transformations, when the ambient complex space is equipped with a volume form.

We will follow the notation adopted in [6]. Small Greek indices run from 1 to n, and the summation convention is used. The Levi form $g_{\alpha\bar{\beta}}$ and its inverse $g^{\bar{\beta}\alpha}$ are used to lower and raise indices, e.g.,

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