J. DIFFERENTIAL GEOMETRY
13 (1978) 11-23

RESIDUES OF SINGULARITIES OF HOLOMORPHIC FOLIATIONS

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1. This note contains an algorithm for the computation of the residues associated with the singularities of holomorphic foliations on compact complex analytic manifolds. We assume that the singular set is a closed holomorphic subvariety, and we drop the requirement, which is essential in [1], [3], that the dimension of the singular subvariety is one less than the dimension of the leaves.

First of all let us briefly review the known results in this direction. Let M be a compact complex analytic manifold of complex dimension n, T the holomorphic tangent bundle, and F a holomorphic vector bundle of fibre dimension k, $1 \leq k \leq n$. Denote by $\underline{T}$ and $\underline{F}$ the sheaves of germs of holomorphic sections of T and F respectively. Suppose that $f\colon F \to T$ is a holomorphic vector bundle map such that: (1) the singular set $\sum$ is a closed holomorphic subvariety of M, (2) $f(F) \mid M - \sum$ is a holomorphic foliation $\mathscr{F}$ of codimension $n-k$, (3) $\dim_c \sum = k - r$, $r \geq 1$, (4) the subsheaf $f(\underline{F})$ of the sheaf $\underline{T}$ is integrable and full. See [1, p. 282]. The integrability guarantees that $\sum$ is a singularity of a foliation on M, and the fullness rules out any unessential singularities. Let ϕ be a symmetric homogeneous polynomial of degree l, $n-k < l \leq n$, in n variables $x_1, \cdots, x_n$, and $\tilde{\phi}$ the unique polynomial in the elementary symmetric functions $\sigma_1, \cdots, \sigma_n$ of $x_1, \cdots, x_n$ such that $\tilde{\phi}(\sigma_1, \cdots, \sigma_l) = (x_1, \cdots, x_n)$. Let $\underline{Q} = \underline{T}/f(\underline{F})$, $c_j(\underline{Q}) =$ the jth Chern class of $\underline{Q}$, and $\phi(\underline{Q}) = \tilde{\phi}(c_1(\underline{Q}), \cdots, c_l(\underline{Q}))$. Then there exists a homology class $\mathrm{Res}_\phi(\mathscr{F}, \sum) \in H_{2n-2l}(\sum; C)$ which depends only on ϕ and on the local behavior of $\mathscr{F}$ near $\sum$, [1]. Moreover, if $\mu_*\colon H_{2n-2l}(\sum; C) \to H^{2l}(M; C)$ is the inclusion followed by the Poincaré duality, $\mu_* \mathrm{Res}_\phi(\mathscr{F}, \sum) = \phi(\underline{Q})$, ([1] and [3] for $k = 1$). One of the basic problems is to compute this class in terms of the "local behavior" of $\mathscr{F}$ near $\sum$. All the results have been obtained ([1], [3]) under the assumption $r = 1$, i.e., $\dim_C \sum + 1 =$ dimension of the leaves of $\mathscr{F}$.

For $r = 1$ and $k = 1$ we have a foliation $\mathscr{F}$ by holomorphic curves with a singularity set $\sum$ being isolated zeros of a holomorphic vector field $X_{\mathscr{F}}$ defining $\mathscr{F}$. If $\lambda_1(p), \cdots, \lambda_n(p)$ are the eigenvalues of the automorphism of T_p, $p \in \sum$, defined by $X_{\mathscr{F}}$, then under the obvious regularity assumptions there

Received November 14, 1975, and, in revised form, August 10, 1976. Supported in part by the National Science Foundation Grant GP-43363.