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# ISOMETRY OF RIEMANNIAN MANIFOLDS TO SPHERES

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## 1. Introduction

Let $M$ be a differentiable connected Riemannian manifold of dimension $n$. We cover $M$ by a system of coordinate neighborhoods $\{U; x^h\}$, where and in the sequel indices $h, i, j, k, \cdots$ run over the range $\{1, 2, \cdots, n\}$, and denote by $g_{ji}$, $\nabla_j$, $K_{kji}{}^h$, $K_{ji}$ and $K$ the metric tensor, the operator of covariant differentiation with respect to the Levi-Civita connection, the curvature tensor, the Ricci tensor and the scalar curvature of $M$ respectively.

An infinitesimal transformation $v^h$ on $M$ is said to be conformal if it satisfies

$$\mathscr{L}_v g_{ji} = \nabla_j v_i + \nabla_i v_j = 2\rho g_{ji} \qquad (v_i = g_{ih} v^h) \tag{1.1}$$

for a certain function $\rho$ on $M$, where $\mathscr{L}_v$ denotes the operator of Lie derivation with respect to the vector field $v$ (see [6]). When we refer in the sequel to an infinitesimal conformal transformation $v$, we always mean by $\rho$ the function appearing in (1.1). When $\rho$ in (1.1) is a constant (respectively, zero), the infinitesimal transformation is said to be homothetic (respectively, isometric).

We also denote by $\mathscr{L}_{D\rho}$ the operator of Lie derivation with respect to the vector field $\rho^i$ defined by

$$\rho^i = g^{ih}\rho_h = \nabla^i \rho , \tag{1.2}$$

where

$$\nabla^i = g^{ih}\nabla_h , \qquad \rho_h = \nabla_h \rho , \tag{1.3}$$

$g^{ih}$ being contravariant components of the metric tensor. We use $g_{ji}$ and $g^{ih}$ to lower and raise the indices respectively.

The problem of finding conditions for a Riemannian manifold admitting an infinitesimal conformal transformation $v$ to be isometric to a sphere has been extensively studied. For the history of this problem, see [7] and [8]. But in almost all the results on this problem the condition $K =$ constant or $\mathscr{L}_v K = 0$ has been assumed. As results in which the conditon $\mathscr{L}_v K = 0$ is not assumed, Sawaki and one of the present authors [12] (see also [11]) proved the following two theorems, in which and the remainder of this section, unless stated

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