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THE DIMENSION OF BASIC SETS

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Let $f: M \to M$ be a C^1 diffeomorphism of a compact connected manifold M. A closed f-invariant set $\Lambda \subset M$ is said to be *hyperbolic* if the tangent bundle of M restricted to Λ is the Whitney sum of two Df-invariant bundles, i.e., if $T_\Lambda M = E^u(\Lambda) \oplus E^s(\Lambda)$, and if there are constants $C > 0$ and $0 < \lambda < 1$ such that

$$|Df^n(V)| \le C\lambda^n |v| \qquad \text{for } v \in E^s,\ n > 0\ ,$$
$$|Df^{-n}(V)| \le C\lambda^n |v| \qquad \text{for } v \in E^u,\ n > 0\ .$$

The diffeomorphism f is said to satisfy *Axiom A* if (a) the non-wandering set $\Omega(f) = \{x \in M: U \cap \bigcup_{n>0} f^n(U) \neq \emptyset$ for every neighborhood U of $x\}$ of f is a hyperbolic set, and (b) $\Omega(f)$ equals the closure of the set of periodic points of f. If f satisfies Axiom A, one has the spectral decomposition theorem of Smale [9] which says $\Omega(f) = \Lambda_1 \cup \cdots \cup \Lambda_l$ where Λ_i are pairwise disjoint, f-invariant closed sets and $f|_{\Lambda_i}$ is topologically transitive.

These Λ_i are called the *basic sets* of f, and it is the object of this article to investigate restrictions on their dimensions imposed by the homotopy type of f and the fiber dimensions of the bundles E^s and E^u. In [11] S. Smale showed that any diffeomorphism can be isotoped to a diffeomorphism satisfying Axiom A with all basic sets of dimension zero. This disproved earlier conjectures that some homotopy classes might contain only diffeomorphisms with a basic set of positive dimension. Theorem 1 below shows that if one restricts either the fiber dimensions of the bundles E^u or the total number of basic sets for f, then there are indeed homotopy classes all of whose diffeomorphisms (subject to these restrictions) have basic sets of positive dimension. In Theorem 2 we investigate diffeomrphisms with a single infinite basic set, the others being isolated periodic orbits. It is a pleasure to acknowledge valuable conversations with R. F. Williams.

We consider diffeomorphisms which in addition to Axiom A satisfy the no-cycle property [10] which we now define. If Λ_i is a basic set of f then its stable and unstable manifolds ([5] or [9]) are defined by

$$W^s(\Lambda_i) = \{x \in M \,|\, d(f^n(x), \Lambda_i) \to 0 \text{ as } n \to \infty\}\ ,$$

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