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LOCAL ISOMETRIC IMBEDDING OF RIEMANNIAN n-MANIFOLDS INTO EUCLIDEAN $(n+1)$-SPACE

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The problem of isometrically imbedding an n-dimensional Riemannian manifold M^n into Euclidean space E^{n+p} has received considerable attention. For example, it is now known that for each n, all infinitely differentiable M^n admit local isometric imbedding into $M^{n+(1/2)n(n+1)}$, and global isometric imbedding into $E^{n+p(n)}$, where $p(n)$ is a certain function whose optimal determination has been the object of recent study.

On the other hand, much less progress has been made in discovering necessary and sufficient conditions for a given M^n to be locally or globally isometrically imbeddable into E^{n+p} for various fixed values of $p < p(n)$. The known results are mostly limited to $p = 0$ and 1. The case $p = 0$ is of course classical—local isometric imbedding of M^n into E^n occurs when the curvature is zero, and global imbedding, when the global holonomy group is trivial. For $p = 1$, many conditions necessary for global imbedding are known, while sufficient conditions must await further local developments. The basic approach here is also classical. Namely, the fundamental theorem for hypersurfaces [2, p. 47] reduces the question of finding necessary and sufficient conditions for local isometric imbedding of M^n into E^{n+1} to the problem of solving the Gauss and Codazzi equations for a suitable second fundamental form tensor, in terms of the curvature tensor of M^n; therefore, the results obtained will necessarily be in the form of conditions on the curvature tensor.

The Gauss and Codazzi equations have been solved by T. Y. Thomas in his fundamental paper [4], and by N. A. Rozenson in her formidable work [3]. Each used different methods and obtained different types of conditions on the curvature tensor. Due to the quite complicated form of these results, however, the local $p = 1$ situation is far from being clear and warrants further work.

In the present paper, we use the method of bivectors and a theorem of W. L. Chow [1] to solve the Gauss (and Codazzi) equations in the case of a nonsingular curvature tensor, getting in this case, new necessary and sufficient conditions for local isometric imbedding of M^n into E^{n+1} (cf. Theorem 4 below).

We proceed with a precise statement of the problem, in our bivector setting. Let V be an n-dimensional real vector space with inner product. Let $\Lambda^2 V$ de-

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