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# INTEGRAL FORMULAS FOR CLOSED SUBMANIFOLDS OF A RIEMANNIAN MANIFOLD

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*Dedicated to Professor Buchin Su on his 57th birthday*

## 1. Introduction

In 1903, H. Minkowski [11] obtained the following two integral formulas for a closed convex surface $S$ in a Euclidean 3-space $E^3$:

$$\int_S (1 + pH)dV = 0 , \quad \int_S (H + pK)dV = 0 , \tag{1.1}$$

where $H$ and $K$ are respectively the mean curvature and the Gaussian curvature of $S$ at a point $P$ whose position vector with respect to the origin 0 of $E^3$ is $x$, $dV$ is the area element of $S$ at $P$, and $p$ is the scalar product $\langle x, e\rangle$ of $x$ and the unit normal vector $e$ of $S$ at $P$. In 1954 C. C. Hsiung [5] extended formulas (1.1) to a closed oriented hypersurface $M^m$ in a Euclidean $(m + 1)$-space $E^{m+1}$ $(m \geq 2)$ and obtained characterizations of hyperspheres in $E^{m+1}$. In 1956 C. C. Hsiung [6] and in 1959 G. F. Feeman and C. C. Hsiung [3] extended Hsiung's integral formulas to the case in which $E^{m+1}$ is a Riemannian space $N^{m+1}$ of constant sectional curvature, and obtained characterizations of umbilical hypersurfaces in $N^{m+1}$. In 1962, Y. Katsurada [7] extended the aforesaid results to a closed oriented hypersurface in $N^{m+1}$ by introducting an infinitesimal conformal vector field $\xi$ to replace the position vector field $x$. In 1968 and 1969, Y. Katsurada, H. Kôjyô and T. Nagai [8], [9], [10] obtained integral formulas for a closed oriented submanifold $M^m$ of dimension $m\,(\geq 2)$ in a Riemannian $n$-manifold $N^n$ $(n > m)$ of constant sectional curvature with respect to an infinitesimal conformal vector field $\xi$ and a special unit normal vector field $e$ of $M^m$, and conditions for $M^m$ to be umbilical with respect to $e$. In 1971 B. Y. Chen and K. Yano [1] studied the case in which the field $e$ is more general but $N^n$ is Euclidean and $\xi$ is the position vector field $x$. The purpose of the present paper is to extend the results of Chen and Yano to the general case in which $N^n$ is Riemannian and $\xi$ is an infinitesimal conformal vector field so that all known results are special cases of ours.

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