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RESIDUES AND CHARACTERISTIC CLASSES FOR RIEMANNIAN FOLIATIONS

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Introduction

In [9] we began a study of characteristic classes for Riemannian foliations. Here we shall continue this study. The basic technique which we will use will be to start with a Killing vector field on R^{4m} with a single nondegenerate singularity at the origin and consider the resulting $(4m-2)$-codimensional Riemannian foliation on a $(4m-1)$-sphere S^{4m-1}. Given an Ad-invariant homogeneous polynomial of degree $2m$ we relate the Bott residue of the vector field determined by this polynomial and the Simons characteristic number associated to the foliation on S^{4m-1} and this polynomial. By mapping various classical groups onto S^{4m-1} and looking at the induced Riemannian foliations, we will obtain infinite classes of examples of families of foliations with trivial normal bundles for which appropriate exotic characteristic classes vary continuously.

As a consequence of these examples we obtain complete results on continuous variation in some of the possible dimensions where variation can occur. Namely, continuous variation does occur for classes in $H^{4m-1}(RW_{4m-2})$ (see § 1). A basis for $H^{4m-1}(RW_{4m-2})$ is given by $p_{j_1}\cdots p_{j_k}h_i$ where $4(j_1+\cdots+j_k)+4i_1-1=4m-1$ and $i\le j_1$ if $k>0$. Call this monomial p_Jh_i. Let $FR\Gamma_q$ denote the fiber of $BR\Gamma_q\to BGL(q)$.

Theorem (3.5). *The map $H^{4m-1}(RW_{4m-2})\to H^{4m-1}(FR\Gamma_{4m-2})$ is injective. The classes p_Jh_i all vary continuously.*

We will also conclude the uncountability of the homotopy groups $\pi_{4m-1}(FR\Gamma_{4m-2})$.

In § 1 we recall some basic facts and establish some notation. In § 2 we prove the basic results relating various connections which will enable us to deduce the relation between the residue and the Simons numbers. In § 3 we exhibit the examples of continuous families of Riemannian foliations with trivial normal bundles on various classical groups for which appropriate exotic classes vary continuously. We also examine the homotopy of $FR\Gamma_q$.

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