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THE STRUCTURE OF SOLUTIONS TO PLATEAU'S PROBLEM IN THE n-SPHERE

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1. Introduction

Let T be a k-dimensional rectifiable current in the unit sphere $\boldsymbol{S}^n$ which is absolutely area minimizing with respect to $\boldsymbol{S}^n$ and is such that ∂T lies in a *closed* m-dimensional geodesic hemisphere Q. We will present results concerning the location of T (Theorems 3.5 and 4.4) and, in case $k = m$, the structure of T (Theorem 4.5). The primary difficulty arises from the assumption that Q is closed; simple examples using lines of longitude on $\boldsymbol{S}^2$ show that not only is T not uniquely determined by ∂T, but there may be a continuum of solutions to the Plateau problem for a fixed boundary lying in Q.

Our results are relevant to the study of the structure of oriented tangent cones at points on the boundary of an area minimizing current in $\boldsymbol{R}^n$ (see [3, 5.2]), and this was our principal motivation for undertaking this study. An application to this problem is given in § 5.

In order to obtain our main results we first prove a "location theorem" for minimizing and minimal (or stationary) currents of arbitrary dimension in $\boldsymbol{S}^n$ which is a formulation for currents in the sphere of the classical idea that a bounded minimal submanifold of $\boldsymbol{R}^n$ must lie in the convex hull of its boundary (a simple proof of which is also given). Such results were first obtained by Blaine Lawson [7] for smooth minimal immersions of manifolds of arbitrary dimension, and for pseudo-immersions in the two dimensional case. We will use his formulation of the notion of convex hull of a subset of $\boldsymbol{S}^n$. The minimal "surfaces" which we consider include Lawson's as special cases; however, because his proofs are centered around use of a maximal principle, his results are stronger than ours.

The author is indebted to his colleague Benjamin Halpern for several stimulating discussions which lead to the use of the function F in the proof of Sheeting lemma 4.4 and Theorem 4.5. This construction has also been recently applied by Sandra Paur in her study of boundary behavior of integral currents [8].

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