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C-TOTALLY REAL SUBMANIFOLDS

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0. Introduction

C. S. Houh [5], S. T. Yau [10], B. Y. Chen and K. Ogiue [3] have studied totally real submanifolds (anti-holomorphic submanifolds) in an almost Hermitian manifold or a Kählerian manifold of constant holomorphic sectional curvature, and obtained many interesting results.

On the other hand, in the recent paper [8] we have investigated the C-totally real submanifolds in a Sasakian manifold with constant ϕ-holomorphic sectional curvature.

In § 1 we recall some basic formulas for submanifolds in Riemannian manifolds. In § 2 we shall state the fundamental property of C-totally real submanifolds in Sasakian manifolds. In the last section, we investigate C-totally real minimal submanifolds M^n in a constant ϕ-holomorphic sectional curvature and show the pinching theorem for the length of the second fundamental form by using the method of J. Simons [7].

1. Preliminaries

Let $\bar{M}$ be a Riemannian manifold of dimension $n + p$, and M an n-dimensional submanifold of $\bar{M}$. Let $\langle\ ,\ \rangle$ be the metric tensor field on $\bar{M}$ as well as the metric induced on M. We denote by $\bar{\nabla}$ the covariant differentiation in $\bar{M}$, and by ∇ the covariant differentiation in M determined by the induced metric on M. Let $\mathfrak{X}(\bar{M})$ (resp. $\mathfrak{X}(M)$) be the Lie algebra of vector fields on $\bar{M}$ (resp. M), and $\mathfrak{X}^{\perp}(M)$ the set of all vector fields normal to M.

The Gauss-Weingarten formulas are given by

$$\begin{aligned} &\bar{\nabla}_X Y = \nabla_X Y + B(X, Y) , \\ &\bar{\nabla}_X N = -A^N(X) + D_X N , \quad X, Y \in \mathfrak{X}(M), N \in \mathfrak{X}^{\perp}(M) , \end{aligned} \tag{1.1}$$

where D is the connection in the normal bundle. Both A and B are called the second fundamental form of M, and satisfy $\langle A^N(X), Y\rangle = \langle B(X, Y), N\rangle$.

The curvature tensors associated with $\bar{\nabla}, \nabla$ and D are defined by

$$\begin{aligned} \bar{R}(X, Y) &= [\bar{\nabla}_X, \bar{\nabla}_Y] - \bar{\nabla}_{[X,Y]} , \\ R(X, Y) &= [\nabla_X, \nabla_Y] - \nabla_{[X,Y]} , \\ R^{\perp}(X, Y) &= [D_X, D_Y] - D_{[X,Y]} . \end{aligned} \tag{1.2}$$

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