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EQUIVALENCE OF STABLE MAPPINGS BETWEEN TWO-DIMENSIONAL MANIFOLDS

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1. Introduction

In this paper, we study stable C^∞ mappings between two-dimensional manifolds. For any stable map $f: M \to N$ we define stratifications of M and N (partitions of M and N into submanifolds, called strata) such that f maps each stratum in M diffeomorphically onto a stratum in N. Suppose f and g are stable maps from M to N, and there exists a homeomorphism $h: M \to M$ inducing a one-to-one correspondence between the stratifications defined on M by f and g such that $(g \circ h)(S) = f(S)$ for each stratum S defined by f. Then there exists a C^∞ diffeomorphism $h': M \to M$ such that $g \circ h' = f$. The above is essentially Theorem 4.1, which is the principle result of this paper and is stated and proved in § 4. The aforementioned stratifications are defined in § 2. In § 3, we give examples illustrating some ways in which Theorem 4.1 cannot be strengthened. In the rest of this section, we make definitions and state known results which will be used later. The material of this paper, except for the proof of Theorem 4.1, is contained in the author's thesis [20].

In [18], C^r mappings ($r \geq 3$) from an open set $U \subseteq \boldsymbol{R}^2$ into $\boldsymbol{R}^2$ were studied by Whitney. Let f be such a mapping, and J its Jacobian matrix. If $\det J(p) \neq 0$, then $p \in U$ is said to be a *regular point* of f; otherwise, p is a *singular point*. Whitney calls p a *good point* for f if p is regular or if $\operatorname{grad}(\det J(p)) \neq 0$. If f is good, that is, each point of U is good for f, then the singular set $\det J = 0$ is a 1-manifold (by the implicit function theorem). Suppose p is a singular point of a good map f with $\varphi(t)$ a regular C^2 parameterization of the singular curve through p. Whitney calls p a *fold point* of f if $d(f \circ \varphi)/dt \neq 0$ at p, and a *cusp point* of f if $d(f \circ \varphi)/dt = 0$ and $d^2(f \circ \varphi)/dt^2 \neq 0$ at p. The definitions of fold and cusp points are independent of the parameterization φ. Cusp points are necessarily isolated. Thus the set F of fold points of f is a 1-manifold; the connected components of F are called *fold curves*. A point p is an *excellent point* of a good map f if it is either regular or else a fold or a cusp point, and f is excellent if each point of U is excellent for f.

If p is a regular point of f, then, by the inverse mapping theorem, C^r coordinate systems (x, y) and (u, v) exist around p and $f(p)$ respectively such that f takes the form $u = x$, $v = y$.

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