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HIGHER ORDER ANALOGUES OF CLASSICAL GROUPS

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1. Introduction

In [2] and [3] one of the present writers introduced a notion of canonical tangential resolution $({}^kM)$ $(k = 0, 1, 2, \cdots)$ for an arbitrary real C^∞ finite-dimensional manifold M. Subsequently, various aspects of the higher-order terms of such a sequence have been investigated (see [4], [5], and [6]). While the local origins of the theory are to be found in the formalism of extensor analysis (see [7] as a general reference), the categorical context is co-equalization in the general theory of cotriples, the basic cotriple being the zero-section and the tangent functor in the category of C^∞ manifolds (see [9]).

The present paper concerns the resolution $({}^kG)$ of a Lie group G and the resolution $({}^k\phi)$ of a differentiable action ϕ of G on a manifold. The principal results are the theorems in § 2 establishing matrix realizations for each ${}^kG$ and its associated Lie algebra $\mathscr{L}({}^kG)$ and interpreting the relevant exponential map in the case where G is a Lie subgroup of some general linear group. The information developed here yields the foundation for a general theory of differentiable fiber bundle resolution and its interpretation, a systematic treatment of which will be given in later papers. The remainder of the introduction explains the notational conventions and special identifications used in the sequel. All manifolds are modeled on real Banach spaces and are at least of class C^∞. The notation is intended to conform as closely as possible with that currently employed in such a context (see [1], [8], and [11]).

Let M be a manifold modeled on the Banach space B. An element of the tangent bundle $T(M)$ will be viewed as an equivalence class $[\theta, b]_x$, where $b \in B$, $x \in M$, and θ is a local coordinate map about x. Thus, if $\phi: M \to N$ is a differentiable map, its associated tangent map $T(\phi): T(M) \to T(N)$ is described locally by

$$T(\phi)([\theta, b]_x) = [\psi, D(\psi \circ \phi \circ \theta^{-1})(\theta(x))b]_y \,, \tag{1}$$

where $y = \phi(x)$, ψ is a local coordinate map about y, and $D(\psi \circ \phi \circ \theta^{-1})(\theta(x))b$ is the total differential of $\psi \circ \phi \circ \theta^{-1}$ at the point $\theta(x)$ evaluated at the vector b. When V is an open set in a Banach space C, $T(V)$ will be viewed as the direct product $V \times C$ with $(v\,; c)$ denoting a tangent vector $c \in C$ located at the point

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