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HARMONIC AND RELATIVELY AFFINE MAPPINGS

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The theory of harmonic mappings of a Riemannian space into another has been initiated by Eells and Sampson [2] and studied by Chern [1], Goldberg [1], [3], T. Ishihara [3], [5] and others.

In this paper, we study projective and affine mappings of a manifold with symmetric affine connection into another and harmonic and relatively affine mappings of a Riemannian space into another.

1. Differentiable mappings of a manifold with symmetric affine connection into another

Let (M, ∇) be a manifold of dimension n with symmetric affine connection ∇, and $(N, \bar{\nabla})$ a manifold of dimension p with symmetric affine connection $\bar{\nabla}$, where $n, p \geq 2$. Let there be given a differentiable mapping $f: M \to N$ which we denote sometimes by $f: (M, \nabla) \to (N, \bar{\nabla})$. Manifolds, mappings and geometric objects which we discuss in this paper are assumed to be of differentiability class C^∞. Take coordinate neighborhoods $\{U; x^h\}$ of M and $\{\bar{U}, y^\alpha\}$ of N in such a way that $f(U) \subset \bar{U}$, where $(x^h) = (x^1, x^2, \cdots, x^n)$ and $(y^\alpha) = (y^{\bar{1}}, y^{\bar{2}}, \cdots, y^{\bar{p}})$ are local coordinates of M and N respectively. The indices $h, i, j, k, l, m, r, s, t$ run over the range $\{1, 2, \cdots, n\}$, and the indices $\alpha, \beta, \gamma, \delta, \lambda, \mu, \nu$ the range $\{\bar{1}, \bar{2}, \cdots, \bar{p}\}$. The summation convention will be used with respect to these two systems of indices. Suppose that $f: (M, \nabla) \to (N, \bar{\nabla})$ is represented by equations

$$y^\alpha = y^\alpha(x^1, x^2, \cdots, x^n) \tag{1.1}$$

with respect to $\{U, x^h\}$ and $\{\bar{U}, y^\alpha\}$. We put

$$A_i^{\ \alpha} = \partial_i y^\alpha(x^1, x^2, \cdots, x^n) \ , \tag{1.2}$$

where $\partial_i = \partial/\partial x^i$. Then the differential df of the mapping f is represented by the matrix $(A_i^{\ \alpha})$ with respect to the local coordinates (x^h) and (y^α) of M and N.

When a function ρ, local or global, is given in N, throughout the paper we shall identify ρ with the function $\rho \circ f$ induced in M. We denote by Γ_{ji}^h the

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