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COMPLETE CONVEX HYPERSURFACES OF A HILBERT SPACE

RUBENS LEÃO DE ANDRADE

1. Statement of the results

A complete *convex hypersurface* M of a Hilbert space H is a one-codimensional C^∞ submanifold of H, which is complete as a metric subspace of H such that $M = \partial K$ is the boundary of a closed convex set K with nonvoid interior. For each $p \in M$ let $\nu(p)$ be the unit normal vector which points to the interior of K. In this way we define the *Gauss map* $\nu : M \to \Sigma$ from M into the unit sphere Σ of H. This is a C^∞ map and its derivative at each point $p \in M$ is self-adjoint. We say that M *bounds a half-line* if there exists a half-line $\{p + tv ; t \geq 0\}$ contained in the interior of K.

In the case where M is a complete convex hypersurface of a Euclidean n-space R^n, the condition for M to bound a half-line is equivalent to that for M to be unbounded. In § 2 we give an example of an unbounded, positively curved, convex hypersurface which does not bound any half-line. In Theorem A we characterize the three possible cases of boundness (bounded, unbounded and bounding a half-line, unbounded and bounding no half-line) in terms of the Gauss map of M. In [5] H. H. Wu proved that if M is an unbounded complete convex hypersurface of R^n such that at a point $p \in M$ the sectional curvatures are all positive, then M is a pseudograph over one of its tangent hyperplanes (see definition below). Our example shows that this is not true in the infinite dimensional case. Theorem B gives a necessary and sufficient condition for M to be a pseudograph over one of its tangent hyperplanes. Theorem C is the Bonnet-Myers theorem for hypersurfaces of a Hilbert space.

In what follows, by a Hilbert space we mean a separable Hilbert space. As usual, int (A) denotes the interior of A and cl (A) its closure.

Theorem A. *Let M be a complete convex hypersurface of a Hilbert space H. Then*:

(1) *M is bounded if and only if the Gauss map* $\nu : M : \to \Sigma$ *is onto,*

(2) *M is unbounded and bounds a half-line if and only if the image of the Gauss map is contained in a hemisphere,*

(3) *M is unbounded and does not bound any half-line if and only if the image of the Gauss map is dense and has void interior.*

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