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RIEMANNIAN SUBMERSIONS WITH TOTALLY GEODESIC FIBERS

RICHARD H. ESCOBALES, JR.

Let M and B be C^∞ Riemannian manifolds. By a *Riemannian submersion* we mean a C^∞ mapping $\pi: M \to B$ from M onto B such that π is of maximal rank and π_* preserves the lengths of horizontal vectors, i.e., vectors orthogonal to the fiber $\pi^{-1}(x)$ for some $x \in B$.

§ 1 is primarily devoted to a summary of known results which will be used in the remaining portion of the paper. § 2 gives a sufficient condition for an isometry f of M to preserve bundle structure where the bundles in question are determined by Riemannian submersions $\pi_i: M \to B$ where M and B are Riemannian manifolds. In this result (Theorem 2.2) we assume M is connected and complete and the fibers are connected and totally geodesic. Now a large class of Riemannian submersions satisfy precisely these conditions. Thus our theorem should have many applications. Some of them (Lemmas 2.4, 2.5 and 2.6) show that many Riemannian submersions from spheres are essentially equivalent to the standard ones (see O'Neill [15], Gray [8]).

In § 3 we classify those B for which there is a Riemannian submersion $\pi: S^m \to B$ where S^m is a sphere and the fibers are connected and totally geodesic. A similar problem for homogeneous sphere bundles was discussed by Nagano in [14]. Since we make no assumption about homogeneity, our proof depends on the properties of submersion metrics.

In differential geometry there has been extensive study of isometric immersions into space forms. Part of Proposition 3.1 together with Theorem 3.4 may be viewed as providing information on the dual question; namely, given a space form what Riemannian submersions are admissible if the fibers are totally geodesic?

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1. In this section we summarize known results on Riemannian submersions

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