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MANIFOLDS WITH HOLONOMY GROUP $Z_2 \oplus Z_2$ AND FIRST BETTI NUMBER ZERO

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In 1957 E. Calabi [1] announced an approach to the classification of flat compact Riemannian manifolds wherein he proved that any n-dimensional flat compact Riemannian manifold with first Betti number q can be constructed from a torus of dimension q and a flat compact Riemannian manifold of dimension $n - q$. The manifolds which are primitive in this approach are therefore those with first Betti number zero. In this paper we construct a rather large family of flat compact Riemannian manifolds with holonomy group isomorphic to $Z_2 \oplus Z_2$, all of which have first Betti number zero. The approach is algebraic, via well-known theorems concerning the classification of flat compact Riemannian manifolds by their fundamental groups.

1. Preliminaries

Let G be an abstract group. We will call G a *space form group of dimension* p if

i. G is torsion-free,
ii. G contains a normal subgroup N_G which is maximal abelian in G,
iii. N_G is free abelian of rank p,
iv. G/N_G is finite.

We will need the following theorems which are all paraphrased from [2, Chapter 3].

Theorem. *An abstract group G is the fundamental group of a flat compact Riemannian manifold of dimension p if and only if G is a space-form group of dimension p. If G is a space-form group, then N_G is uniquely determined and the holonomy group of the manifold is isomorphic to G/N_G.*

Theorem. *Two flat compact Riemannian manifolds are affinely equivalent if and only if they have isomorphic fundamental groups.*

We fix the following notation. Let n be a positive integer. A_n will denote a fixed set of cardinality $4n$, i.e., $A_n = \{a_1, \cdots, a_{4n}\}$. P_n will denote the group of permutations of the set A_n. (I.e., P_n is isomorphic to the symmetric group on $4n$ symbols.) T_n will denote the free abelian group generated by the set A_n. Finally, E_n will denote the semi-direct product of T_n with P_n via the obvious action of P_n on T_n.

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