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GENERALIZED SCALAR CURVATURES OF COHOMOLOGICAL EINSTEIN KAEHLER MANIFOLDS

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1. Introduction

In Riemannian geometry all elementary symmetric polynomials of eigenvalues of the Ricci tensor are geometric invariants. In particular, the one of degree 1 is called the scalar curvature.

In this paper, we shall study some properties of the geometric invariants for *cohomological Einstein* Kaehler manifolds. Let M be a Kaehler manifold with fundamental 2-form Φ and Ricci 2-form γ. We say that M is cohomologically Einsteinian if $[\gamma] = a\cdot[\Phi]$ for some constant a, where $[*]$ denotes the cohomology class represented by $*$. It is well-known that the first Chern class $c_1(M)$ is represented by γ.

Let $z_1, \cdots, z_n$ be a local coordinate system in M, $g = \sum g_{\alpha\bar\beta} dz_\alpha d\bar z_\beta$ be the Kaehler metric of M, and $S = \sum R_{\alpha\bar\beta} dz_\alpha d\bar z_\beta$ be the Ricci tensor of M. Define n scalars $\rho_1, \cdots, \rho_n$ by

$$\frac{\det(g_{\alpha\bar\beta} + tR_{\alpha\bar\beta})}{\det(g_{\alpha\bar\beta})} = 1 + \sum_{k=1}^{n} \rho_k t^k ,$$

and denote the scalar curvature of M by ρ. Then it is easily seen that $\rho = 2\rho_1$, and is also clear that $\rho_n = \det(R_{\alpha\bar\beta})/\det(g_{\alpha\bar\beta})$.

We shall prove

Theorem 1. *Let M be an n-dimensional compact cohomological Einstein Kaehler manifold. If $c_1(M) = a\cdot[\Phi]$, then*

$$\int_M \rho_k * 1 = (2\pi a)^k \binom{n}{k} \int_M * 1 ,$$

where $\binom{n}{k}$ denotes the binomial coefficient, and $*1$ the volume element of M.

This results implies that the average of ρ_k, $\int_M \rho_k * 1 \Big/ \int_M *1$, does not depend on the metric too strongly.

Let $P_{n+p}(C)$ be an $(n+p)$-dimensional complex projective space with the

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