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THE FRENET FRAME OF AN IMMERSION

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Introduction

It is a known theorem of Jacobi that the indicatrix of the principal normal of a curve in a Euclidean three-space E^3 divides the unit sphere S^2 into two pieces of equal area. In this paper a generalization of this theorem is given in the sense that the curve is replaced by a two-sphere S^2 imbedded in a Euclidean 4-space E^4.

To define a principal normal of an immersion $x: M^n \rightarrow E^{n+N}$ of a manifold M^n into a Euclidean space E^{n+N} we proceed as follows. If we take the boundary of a small tubular neighborhood of a curve in the three-space and examine the maximal value of the Gaussian curvature along a fiber over a fixed point of the curve, then the point of the boundary of the tubular neighborhood, at which the Gaussian curvature attains its maximal value, together with the fixed point of the curve defines the principal normal of the curve. This construction can be generalized to a manifold M^n immersed in E^{n+N} by replacing the tubular neighborhood by the normal bundle B_ν of the immersion and the Gauss curvature by the Killing-Lipschitz curvature as defined in [2], and the invariant local cross sections in B_ν thus obtained are called the Frenet frame of the immersion x. These cross sections enable us to define in an obvious way also local invariant cross sections in the tangent bundle B_τ. However we shall not need them in this paper, and therefore their construction will be omitted. For $n = 2$, $N = 2$ the construction of a Frenet frame in our sense was given by T. Ōtsuki in [5].

The construction of a Frenet frame leads to the definition of new invariants of the immersed manifold $x(M^n)$ called mixed curvatures, by means of which we can generalize to closed even-dimensional manifolds the K. Borsuk's theorem [1] concerning the total curvature of a closed curve in a Euclidean n-space, $n \geq 3$.

Furthermore, we give another proof of a result of D. Ferus [4] concerning the total curvature of a knotted sphere of codimension two imbedded in a Euclidean space.

In this paper all manifods and mappings are supposed to be of class C^∞.

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