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A GENERALIZED ALLENDOERFFER-WEIL FORMULA AND AN INEQUALITY OF THE COHN-VOSSEN TYPE

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1. Introduction

In this paper we present a version of the Gauss-Bonnet-Chern formula which applies to arbitrary compact locally convex subsets C of a riemannian manifold M. The classical counterpart is the Allendoerffer-Weil formula for riemannian polyhedra [1]. But while the singularities of a polyhedron are separated along submanifolds, we do not have to restrict the set of singularities, in particular, this set can be dense in the boundary of C.

An important role is played in our formula by the set $\mathscr{N}_C$ of outer vectors of C, which is a locally lipschitz submanifold of T_1M. In fact, the boundary terms appear as integrals over $\mathscr{N}_C$ of an almost everywhere defined differential form, in which enter curvature quantities of M along ∂C and a generalized second fundamental form of ∂C which is symmetric and positive semidefinite almost everywhere.

In the case of a positive semidefinite curvature operator along ∂C, the boundary terms can be estimated thus giving a sort of Cohn-Vossen inequality for such sets C. For dimensions not exceeding 6, the assumption on the curvature operator can be replaced by $K \geq 0$ along ∂C. Several corollaries apply to the existence of, and bounds for, the total curvature of complete manifolds of nonnegative curvature.

2. Preliminaries

Let $(M, \langle\ ,\ \rangle)$ always be a smooth oriented and connected riemannian manifold of dimension $m \geq 2$. In any case, all manifolds are supposed to be Hausdorff and paracompact. The general notation for M is the same as in [18].

2.1. If $f: N \to M$ is smooth, by a *form A of bidegree (r, s) along f* we mean an alternating form on N of degree r with values in $\bigwedge^s f^*TM$ [2, § 8.3]. For such forms there are the usual algebraic operations $+, \wedge$ and, by the given orientation of M, every form Q of bidegree (r, m) corresponds to a form $[Q]$ of bidegree $(r, 0)$, i.e., to a real r-form on N. If $g: N_1 \to N$ is smooth, then for every given A there is defined a form g^*A of the same bidegree along $f \circ g$. The operation g^* is homomorphic with respect to $+, \wedge, [\ ]$, and for another

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