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RIEMANNIAN MANIFOLDS ADMITTING AN INFINITESIMAL CONFORMAL TRANSFORMATION

KENTARO YANO & HITOSI HIRAMATU

1. Introduction

Let M be an n-dimensional connected Riemannian manifold with positive definite metric of differentiability class C^∞. We cover M by a system of coordinate neighborhoods $\{U; x^h\}$, and denote by $g_{ji}, \nabla_i, K_{kji}{}^h, K_{ji}$ and K the fundamental metric tensor field, the operator of covariant differentiation with respect to the Levi-Civita connection, the curvature tensor field, the Ricci tensor field and the scalar curvature field of M respectively. Here and in the sequel indices $h, i, j, k, \cdots$ run over the range $\{1, \cdots, n\}$.

We denote by $C_0(M)$ the largest connected group of conformal transformations of a Riemannian manifold M, and by $I_0(M)$ the largest connected group of isometries of M.

Riemannian manifolds with constant scalar curvature field admitting an infinitesimal nonhomothetic conformal transformation have been extensively studied and we know the following theorems.

Theorem A (*Yano and Nagano* [38]). *If M is a complete Einstein manifold of dimension $n > 2$ and*

$$C_0(M) \neq I_0(M) , \tag{1.1}$$

then M is isometric to a sphere.

(See also Bishop and Goldberg [3].)

Theorem B (*Nagano* [23]). *If M is a complete Riemannian manifold of dimension $n > 2$ with parallel Ricci tensor field and* (1.1) *holds, then M is isometric to a sphere.*

Theorem C (*Goldberg and Kobayashi* [5], [6], [7]). *If M is a compact homogeneous Riemannian manifold of dimension $n > 3$, and* (1.1) *holds, then M is isometric to a sphere.*

Theorem D (*Lichnerowicz* [22]). *If M is a compact Riemannian manifold of dimension $n > 2$, $K =$ const., and $K_{ji}K^{ji} =$ const., then* (1.1) *implies that M is isometric to a sphere.*

Theorem E (*Hsiung* [11], [12], [13]). *If M is compact and of dimension*

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