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# ON CHERN'S KINEMATIC FORMULA IN INTEGRAL GEOMETRY

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*Dedicated to S. S. Chern on his 60th birthday*

## 1. Introduction

In 1939 Hermann Weyl [1] derived a formula for the volume of the tube of radius $\rho$ about a compact manifold (without boundary) imbedded in a Euclidean space. The expression for this volume, for a manifold $X$ of dimension $k$ imbedded in a Euclidean $n$-space $E^n$ is a polynomial $V(T_\rho^{(n)}(X))$ in $\rho$, valid for small $\rho$, when no self-intersections in the normal bundle occur. The coefficients of this polynomial are integrals over $X$ of invariant polynomial functions of the Riemann-Chistoffel curvature tensor. The polynomial expression for the volume is of the form

$$V(T_\rho^{(n)}(X)) = \sum \gamma_{n,k,e}\mu_e(X)\rho^{n-k-e} , \tag{1.1}$$

where the summation extends over all even values of $e$ such that $0 \leq e \leq k$. The $\mu_e(X)$ are the integral invariants referred to, while the $\gamma_{n,k,e}$ depend only on their subscripts and not on more subtle geometric properties of $X$. Thus $\gamma$ and $\mu$ are uniquely determined up to a factor which depends on $k$ and $e$. In what follows we add a superscript (1) to $\mu$ when quoting others.

In 1966 S. S. Chern [2] studied the same $\mu$'s from the point of view of the kinematic formula. Let $M^p$ and $M^q$ be compact manifolds of dimensions $p$ and $q$ imbedded in $E^n$, and let $g$ be an element of the group of isometries in $E^n$. Then, for almost all $g$, $M^p \cap gM^q$ is again a manifold, and the $\mu_e^{(1)}(M^p \cap gM^q)$ are meaningful quantities. The kinematic formula of Chern deals with the integral $\int \mu_e^{(1)}(M^p \cap gM^q)d^{(1)}g$, where the integration extends over the group of isometries, and $d^{(1)}g$ is the Haar measure on this group, i.e., the product of the measure on $E^n$ and that on the orthogonal group in $n$ dimensions, the latter being a product of measures on spheres. This integral, according to Chern's theorem, is expressible as follows:

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