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MANIFOLDS WITH REFLECTING BOUNDARY

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1. Introduction

Let M be a compact oriented C^∞ manifold of dimension $4k$ with boundary B of dimension $4k - 1$, and let g be a Riemannian metric on M. Being given a power series with real coefficients $P \in R[[x_{4i}]]$ in variables $x_{4i}, i = 1, 2, \cdots$ one may replace x_{4i} by the real $4i$-form P_i which gives the i-th Pontrjagin class $\mathscr{P}_i$ of M, expressed in the standard way in terms of the curvature 2-forms Ω_{jk} of the Riemannian metric g, and integrate the component of dimension $4k$ over M to obtain a real number

$$P(M, g) = \int_M P(P_1, P_2, \cdots) .$$

If M is closed, *i.e.*, B is empty, then $P(M, g)$ is independent of the Riemannian metric g, and many be obtained by replacing x_{4i} by the i-th Pontrjagin class $\mathscr{P}_i$ of M, and evaluating the 4k-dimensional component of the resulting cohomology class on the fundamental homology class of M; *i.e.*,

$$P(M, g) = \langle P(\mathscr{P}_1, \mathscr{P}_2, \cdots), [M] \rangle .$$

In [5], C. C. Hsiung has introduced another class of manifolds for which these numbers are well behaved, which he calls manifolds with reflecting boundary. Specifically, one considers a manifold M together with an orientation reversing involution $\pi: B \to B$. For such a pair (M, π) one considers a "nice" Riemannian metric g on M, which satisfies the conditions that π is an isometry of the manifold B with induced Riemannian metric g/B and that, on a tubular neighborhood $B \times [0, 1)$ of $B = B \times 0$ in M, g is given by a product metric. Such metrics always exist.

Proposition 1. *If (M, π) is a manifold with reflecting boundary with nice Riemannian metric g, then $P(M, g)$ is independent of the nice metric. Further,*

a) *if $P \in Z[[x_{4i}]]$ is a power series with integral coefficients, then $P(M, g)$ belongs to $\frac{1}{2}Z$,*

b) *if P is a power series of the form $Q \cdot L$ where Q, L are the rational power series given by considering x_{4i} as the i-th elementary symmetric function in variables y_j (of dimension 2), with Q any symmetric polynomial over the integers in the variables $e^{y_j} + e^{-y_j} - 2$ and with L the product of the classes*

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