J. DIFFERENTIAL GEOMETRY
9 (1974) 219-238

CONTACT MANIFOLDS

R. E. STONG

1. Introduction

This paper is a study of the differential topology of contact manifolds. In § 2, the known results on contact manifolds will be reviewed. The problem of existence of a global contact form will be studied, straightening out a minor error in Gray [4]. The reduction of the structure group of the tangent bundle will be examined. In § 3, the Kervaire semi-characteristic will be defined and studied. Classically, there are two types of contact manifolds: a) the bundle of tangent corays to a closed manifold and b) the spheres. The semi-characteristic distinguishes these types. In § 4, the semi-characteristic will be placed in a cobordism framework. This provides insight into just what the invariant measures. In § 5, it will be shown that a closed oriented contact manifold of dimension $8k + 5$ is the boundary of a compact almost complex manifold. In § 6, the characteristic classes of contact manifolds will be studied.

The author is indebted to Professor E. E. Floyd for some helpful conversations and to the National Science Foundation for financial support during this work.

2. Review of contact manifolds

Let $(x^1, \cdots, x^n, y^1, \cdots, y^n, z)$ be coordinates in Euclidean $(2n + 1)$-space R^{2n+1}, and let α_0 be the 1-form on R^{2n+1} defined by $\alpha_0 = dz - \sum y^i dx^i$. This form is completely characterized by the fact that $\alpha_0 \wedge (d\alpha_0)^n \neq 0$ in the sense that any form with this property has the given expression in suitably chosen local coordinates. A diffeomorphism $f\colon U \to V$ between open sets of R^{2n+1} is called a *contact transformation* if $f^*\alpha_0 = \rho\alpha_0$ for some nonzero real function ρ on U. The collection Γ of all contact transformations forms a pseudogroup.

The systematic study of pseudogroups began with the work of Sophus Lie on transformation groups [9], and volume two of his work is devoted to the study of contact transformations.

An odd dimensional manifold M^{2n+1} is called a *contact manifold* if there is an open cover $\{U_i\}$ of M with homeomorphisms $f_i\colon U_i \to V_i \subset R^{2n+1}$ so that when $f_i \cdot f_j^{-1}$ is defined, $f_i \circ f_j^{-1} \in \Gamma$. Two such systems $\{U_i, f_i\}, \{U'_i, f'_i\}$ are equi-

Communicated by S. S. Chern, October 3, 1972.