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ALMOST COTANGENT MANIFOLDS

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1. The geometry of the cotangent manifold $T^*\mathscr{M}$ of a differentiable manifold $\mathscr{M}$ has been studied by K. Yano and E. M. Patterson [4], [5], [6]. Some of their results can be extended to a manifold M of dimension $2n$ carrying a G-structure whose group consists of all $2n \times 2n$ matrices of the form

$$\begin{bmatrix} A & 0 \\ B & (A^{-1})^t \end{bmatrix} \tag{1.1}$$

where $A \in GL(R^n)$ and $A^tB = B^tA$. Such a structure is an *almost cotangent structure,* and such a manifold M is an *almost cotangent manifold* (M. R. Bruckheimer [1]).

Example 1.1. Suppose that $\mathscr{M}$ is a manifold of dimension n, and that $\pi\colon T^*\mathscr{M} \to \mathscr{M}$ is the natural projection which takes a covector at $m \in \mathscr{M}$ to the point m. Any function f in $\mathscr{M}$ can be lifted to a function $f \circ \pi$ in $T^*\mathscr{M}$ but we shall denote it by the same symbol f. If x is a chart of $\mathscr{M}$ with domain V, we can define a standard chart (x, y) of $T^*\mathscr{M}$ with domain $\pi^{-1}V$. Two such charts (x, y), $(\bar{x}, \bar{y})$ with intersecting domains are related by a change of coordinates whose Jacobian matrix has the form (1.1) with

$$A = \left[\frac{\partial x^a}{\partial \bar{x}^b}\right], \qquad B = \left[\frac{\partial^2 \bar{x}^c}{\partial x^a \partial x^d} \frac{\partial x^d}{\partial \bar{x}^b} \bar{y}_c\right], \tag{1.2}$$

where $a, b, c, d = 1, \cdots, n$. The natural moving frames associated with these charts therefore define an almost cotangent structure on $T^*\mathscr{M}$.

Suppose that M is any almost cotangent manifold. We define a 2-form ω on M by specifying its components to be

$$\begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} \tag{1.3}$$

relative to any adapted frame of M. ω determines an *almost symplectic structure* on M to which the given almost cotangent structure is subordinate. If $(\theta^1, \cdots, \theta^{2n})$ is any adapted moving coframe of M, then locally

$$\omega = \theta^a \wedge \theta^{a+n} \qquad (a = 1, \cdots, n) .$$

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