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POISSON COMPLEXES AND SUBELLIPTICITY

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0. Introduction

Let

$$E^0 \xrightarrow{D} E^1 \xrightarrow{D} \cdots \xrightarrow{D} E^N \tag{0.1}$$

be a complex of first-order differential operators on a (compact) n-dimensional manifold X, and let

$$0 \to E^0 \xrightarrow{\sigma(D)(x,\zeta)} E^1 \xrightarrow{\sigma(D)(x,\zeta)} \cdots \xrightarrow{\sigma(D)(x,\zeta)} E^N \to 0 \tag{0.2}$$

be the associated top-symbol complex on $T^*X/\{0\}$. If the bundles E^i are given Hermitian metrics we may define the adjoint operators D^*.

Definition 0.1. We say that (0.1) is $\frac{1}{2}$-subelliptic at position i if and only if an estimate

$$\|u\|_{1/2} \leq C_K\{\|D_i u\|_0 + \|D^*_{i-1}u\|_0 + \|u\|_0\} \,, \quad u \in C_0^\infty(K, E^i) \tag{0.3}$$

holds for each compact subset $K \subseteq X$. Here the norms are Sobolev norms.

Hörmander [7] showed that the estimate (0.3) depends only on the behavior of the top-symbol complex (0.2) in the neighborhood of the characteristic variety, and, in fact, is equivalent to certain "test-estimates" at each point of the characteristic variety.

Guillemin [4], [5], by introducing the notion of asymptotic derivative, was able to reformulate the test-estimates of Hörmander as L^2-exactness of certain asymptotic test-complexes associated to (0.2). Using this new formulation of $\frac{1}{2}$-subellipticity he was able to show that $\frac{1}{2}$-subellipticity is independent of the choice of Hermitian metrics for the bundles E^i (despite the fact that the adjoint operators D^* are defined in terms of the Hermitian metrics), and that $\frac{1}{2}$-subellipticity is invariant under formal pseudo-differential conjugations of the origi-

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