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UMBILICAL SUBMANIFOLDS WITH RESPECT TO A NONPARALLEL NORMAL DIRECTION

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Let M^n be an n-dimensional submanifolds[1] of an $(n+2)$-dimensional euclidean space E^{n+2}, and C be a unit normal vector field of M^n in E^{n+2}. If the second fundamental tensor in the normal direction C is proportional to the first fundamental tensor of the submanifold M^n, then M^n is said to be *umbilical* with respect to the normal direction C. The normal direction C is said to be *parallel* if the covariant differentiation of C along M^n has no normal component, and C is said to be *nonparallel* if the covariant differentiation of C along M^n has nonzero normal component everywhere.

In a previous paper [1], the authors proved that a submanifold is umbilical with respect to a parallel normal direction C if and only if it is contained either in a hypersphere or in a hyperplane of the euclidean space. In the present paper, we shall study the submanifolds of codimension 2 of a euclidean space which are umbilical with respect to a nonparallel normal direction.

1. Preliminaries

We consider a submanifold M^n of codimension 2 of an $(n+2)$-dimensional euclidean space E^{n+2}, and represent it by

$$X = X(\xi^1, \cdots, \xi^n) , \tag{1}$$

where X is the position vector from the origin of E^{n+2} to a point of the submanifold M^n, and $\{\xi^h\}$ is a local coordinate system in M^n, where and throughout this paper the indices $h, i, j, k, \cdots$ run over the range $\{1, \cdots, n\}$.

Put

$$X_i = \partial_i X , \qquad \partial_i = \partial / \partial \xi^i , \tag{2}$$

and denote by C and D two mutually orthogonal unit normals to M^n. Then, denoting by ∇_j the operator of covariant differentiation with respect to the Riemannian metric $g_{ji} = X_j \cdot X_i$ of M^n, we have the equations of Gauss

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[1] Manifolds, mappings, functions, ... are assumed to be sufficiently differentiable, and we shall restrict discussions only to manifolds of dimension $n > 2$.