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AN AFFINE CONNECTION IN AN ALMOST QUATERNION MANIFOLD

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0. Introduction

The theory of differential concomitants was initiated by Schouten [4] and developed by Frölicher [1] and Nijenhuis [1], [2]. It is now well known that for two given tensor fields F and G of type $(1, 1)$ in a differentiable manifold the expression

$$(0.1)\qquad \begin{aligned}[F, G](X, Y) = [FX, GY] - F[X, GY] - G[FX, Y] + [GX, FY]\\ - G[X, FY] - F[GX, Y] + (FG + GF)[X, Y]\end{aligned}$$

defines a tensor field $[F, G]$ of type $(1, 2)$, and that the tensor field $[F, F]$ plays a very important role in the discussion of integrability conditions of an almost complex structure defined by F. We call $[F, G]$ defined by (0.1) the Nijenhuis tensor of F and G.

The present authors [9] proved that if the tensor fields F and G of type $(1, 1)$ satisfy $FG = GF$, then the expression

$$(0.2)\qquad [FX, GY] - F[X, GY] - G[FX, Y] + FG[X, Y]$$

defines a tensor field of type $(1, 2)$. If F and G satisfy $FG + GF = 0$, then the expression (0.1) takes the form

$$(0.3)\qquad \begin{aligned}[F, G](X, Y) = [FX, GY] - F[X, GY] - G[FX, Y]\\ + [GX, FY] - G[X, FY] - F[GX, Y] .\end{aligned}$$

All these tensor fields of type $(1, 2)$ contain F, G and partial derivatives of F, G of the first order.

On the other hand, Walker [6] found a tensor field of type $(1, 4)$ formed with an almost complex structure F, which contains F and partial derivatives of F of the first and the second orders (see also Willmore [7]). Ślebodziński [5] announced that he obtained a tensor field of type $(1, 3)$ formed with an almost complex structure F containing F and partial derivatives of F of the

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