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CONDITIONS FOR CONSTANCY OF THE HOLOMORPHIC SECTIONAL CURVATURE

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In the present note we shall first prove an algebraic result (Theorem 1) on the curvature tensor of a Kaehlerian manifold. As applications we derive two results (Theorems 2 and 3) characterizing constancy of the holomorphic sectional curvature by the existence of sufficiently many complex or totally real submanifolds which are totally geodesic. A special case of Theorem 2 has been known as the axiom of holomorphic planes [3].

1. Curvature tensor

Let M be a Kaehlerian manifold. In the tangent space at a point we consider the curvature tensor R, the complex structure J, and the inner product $\langle\ ,\ \rangle$ arising from the Kaehlerian metric of M. We have $\langle Jx, Jy\rangle = \langle x, y\rangle$ for any two vectors x and y. In addition to the usual properties of the curvature tensor of a Riemannian manifold, R possesses the following properties:

$$R(x, y)J = JR(x, y)\ , \tag{1}$$

$$R(Jx, Jy) = R(x, y)\ . \tag{2}$$

A subspace S of the tangent space is holomorphic if $J(S) = S$. S is said to be *totally real* if it satisfies the following condition:

$$\langle Jx, y\rangle = 0 \qquad \text{for all } x, y \in S\ . \tag{$*$}$$

If P is a 2-dimensional subspace, with an orthonormal basis $\{x, y\}$, of the tangent space, then the sectional curvature $k(P)$ is given by $\langle R(x, y)y, x\rangle$. If P is holomorphic, then the holomorphic sectional curvature $k(P)$ is equal to $\langle R(x, Jx)Jx, x\rangle$, where x is an arbitrary unit vector in P. It is well known (for example, see [1, p. 167]) that $k(P)$ is equal to a constant c for all holomorphic planes P if and only if R is of the form

$$R_c(x, y) = \tfrac{1}{4}c(x \wedge y + Jx \wedge Jy + 2\langle x, Jy\rangle J)\ , \tag{3}$$

where, in general, $x \wedge y$ denotes the endomorphism which maps z into $\langle y, z\rangle x - \langle x, z\rangle y$.

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