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ISOTROPIC IMMERSIONS

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1. Introduction

A Kaehler manifold of constant holomorphic sectional curvature is called a *complex space form*. A *Kaehler immersion* is an isometric immersion which is complex analytic. The second named author proved the following results.

Proposition 1 [2]. *Let M be an n-dimensional complex space form of constant holomorphic sectional curvature c, and $\tilde{M}$ be an $(n + p)$-dimensional complex space form of constant holomorphic sectional curvature $\tilde{c}$. If M is a Kaehler submanifold of $\tilde{M}$ with parallel second fundamental form, then either $c = \tilde{c}$ (i.e., M is totally geodesic in $\tilde{M}$) or $c = \frac{1}{2}\tilde{c}$, the latter case arising only when $\tilde{c} > 0$. Moreover, the immersion is rigid.*

Proposition 2 [3]. *Let M be an n-dimensional complex space form of constant holomorphic sectional curvature c, and $\tilde{M}$ be an $(n + \frac{1}{2}n(n + 1))$-dimensional complex space form of constant holomorphic sectional curvature $\tilde{c}$. If M is a Kaehler submanifold of $\tilde{M}$, then either $c = \tilde{c}$ (i.e., M is totally geodesic in $\tilde{M}$) or $c = \frac{1}{2}\tilde{c}$, the latter case arising only when $\tilde{c} > 0$. Moreover, the immersion is rigid.*

In the present paper, we shall prove similar results for real manifolds. An *isotropic immersion* is an isometric immersion such that all its normal curvature vectors have the same length at each point. A Riemannian manifold of constant curvature is called a *space form*.

Theorem 1. *Let M be an n-dimensional space form of constant curvature c, and $\tilde{M}$ be an $(n + \frac{1}{2}n(n + 1) - 1)$-dimensional space form of constant curvature $\tilde{c}$. If $c < \tilde{c}$, and M is an isotropic submanifold of $\tilde{M}$ with parallel second fundamental form, then $c = \dfrac{n}{2(n + 1)}\tilde{c}$, and the immersion is rigid.*

Theorem 2. *Let M be an n-dimensional space form of constant curvature c, and $\tilde{M}$ be an $(n + \frac{1}{2}n(n + 1) - 1)$-dimensional space form of constant curvature $\tilde{c}$. If $c < \tilde{c}$, and M is an isotropic submanifold of $\tilde{M}$, then $c = \dfrac{n}{2(n + 1)}\tilde{c}$, and the immersion is rigid provided that $n \leq 4$.*

Remark. Theorems 1 and 2 give a (local) characterization of a *Veronese manifold*.

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