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DIFFERENTIAL GEOMETRY OF $S^n \times S^n$

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To Shoshichi Kobayashi on his fortieth birthday

0. Introduction

Blair [1], $\cdots$, [5], Eum [9], Ishihara [10], Ki [9], [11], [12], Ludden [1], $\cdots$, [5], Okumura [13], [14], [15] and the present author [2], $\cdots$, [5], [7], $\cdots$, [15] started the study of the structures induced on submanifolds of codimension 2 of an almost Hermitian manifold or on hypersurfaces of an almost contact metric manifold. Okumura and the present author called these structures (f, g, u, v, λ)-structures, where f is a tensor field of type $(1, 1)$, g a Riemannian metric, u and v 1-forms, and λ a function satisfying

$$\begin{gathered} f^2 = -1 + u \otimes U + v \otimes V , \\ u \circ f = \lambda v , \quad v \circ f = -\lambda u , \quad fU = -\lambda V , \quad fV = \lambda U , \\ u(U) = 1 - \lambda^2 , \quad u(V) = 0 , \quad v(V) = 1 - \lambda^2 , \\ g(fX, fY) = g(X, Y) - u(X)u(Y) - v(X)v(Y) \end{gathered}$$

for arbitrary vector fields X and Y, where U and V are vector fields associated with 1-forms u and v respectively.

An (f, g, u, v, λ)-structure is said to be normal if it satisfies $S = 0$ where S is a tensor field of type $(1, 2)$ defined by

$$S(X, Y) = N(X, Y) + (du)(X, Y)U + (dv)(X, Y)V$$

for arbitrary vector fields X and Y, N being the Nijenhuis tensor formed with f.

A typical example of a differentiable manifold with a normal (f, g, u, v, λ)-structure is an even-dimensional sphere S^{2n}. Ki [11], [12], Okumura [14] and the present author [11], [12], [14] obtained some characterizations of an even-dimensional sphere from this point of view.

The product $S^n \times S^n$ of two spheres of the same radius and the same dimension is also an example of a differentiable manifold with an (f, g, u, v, λ)-structure, but the structure is not normal. Blair [3], [5], Ishihara [10], Ludden

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