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THE MEAN CURVATURE FOR p-PLANE

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Introduction

Let M be an n-dimensional Riemannian space. For the skew symmetric tensor $u_{\lambda_1\cdots\lambda_p}$, $F_p(u)$ for $p = 1, \cdots, n$ are defined as follows:

$$F_1(u) = R_{\lambda\mu}u^\lambda u^\mu \ ,$$

$$F_p(u) = R_{\lambda\mu}u^{\lambda\rho_2\cdots\rho_p}u^\mu{}_{\rho_2\cdots\rho_p} + \frac{p-1}{2}R_{\lambda\mu\nu\omega}u^{\lambda\mu\rho_3\cdots\rho_p}u^{\nu\omega}{}_{\rho_3\cdots\rho_p} \ , \qquad p \geq 2 \ ,$$

where $R_{\lambda\mu\nu\omega}$ is the Riemannian curvature tensor and $R_{\lambda\mu} = R_{\alpha\lambda\mu}{}^\alpha$ is the Ricci tensor of M. Throughout this paper indices $\lambda, \mu, \nu, \cdots$ range from 1 to n, tensors and vectors will be represented with respect to the natural basis unless stated otherwise, and the summation convention is assumed for these indices. Concerning $F_p(u)$ the following theorems are known.

Theorem A [5, p. 64], [3]. *If the quadratic form $F_p(u)$ is positive definite in a compact Riemannian space, there exists no harmonic p-form other than the zero form.*

Theorem B [5, p. 67]. *If $F_p(u)$ is negative definite in a compact Riemannian space, there exists no Killing tensor field of degree p other than the zero tensor.*

Theorem C [4], [2]. *If $F_p(u)$ is negative definite in a compact Riemannian space for $p \leq n/2$, there exists no conformal Killing tensor field of degree p other than the zero tensor.*

In this paper in § 2 we shall give a geometric meaning of $F_p(u)$ in terms of the sectional curvature for a special form u to be called a simple form u, and § 3 is devoted to the discussion of the spaces in which $F_p(u)$ is independent of the simple form u.

1. Preliminaries

Let M be an n-dimensional Riemannian space. Consider a pair of orthonormal vectors $X = (X^\lambda)$ and $Y = (Y^\lambda)$ at a point $m \in M$. Then the sectional curvature of the 2-plane spanned by X and Y is given by

$$\rho(X, Y) = -R_{\lambda\mu\nu\omega}X^\lambda Y^\mu X^\nu Y^\omega \ .$$

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