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A CONDITION FOR PARACOMPACTNESS OF A MANIFOLD

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1. Introduction

It is known that if a differential manifold M is paracompact, then it can be made into a Riemannian manifold with a unique torsion-free Levi-Civita connection. In discussing the structure of Minkowski spaces (see [5]), the author came across a condition for paracompactness of a manifold. This condition is stated and proved as Theorem 1, which is the main result of this paper. We begin by introducing some geometrical preliminaries.

2. Geometrical preliminaries

By a differentiable n-dimensional manifold of class C^r, we mean a Hausdorff connected locally Euclidean topological space with a fixed C^r atlas. We assume r to be large enough to ensure the smoothness of the operations involved. By a pseudo-Riemannian manifold, we mean a manifold with fundamental tensor of arbitrary signature (definite or indefinite). Let L denote the principal fibre bundle of linear frames on M with structure group $G = GL(n, R)$, and let H be the closed subgroup of G which leaves a given nondegenerate quadratic form on $\boldsymbol{R}^n$ invariant. Expressing $x \in \boldsymbol{R}^n$ in terms of its natural basis, we can write the quadratic form $Q: R^n \to R$ as

$$Q(x) = a_{ij}x^i x^j ,$$

where $x = (x^1, \cdots, x^n) \in \boldsymbol{R}^n$, $a_{ij} \in R$, and summation convention is used. Consider the action of G on $L \times G/H$, given by

$$a\cdot(l, \xi) = (a\cdot l, \xi\cdot a^{-1}) \in L \times G/H$$

for $a \in G$ and $(l, \xi) \in L \times G/H$, where a acts on the frame l by acting on each vector in the frame and G/H is regarded as a right coset space.

The quotient space of $L \times G/H$ under this action of G is denoted by $E(M, G/H, G, L)$ or E for short. The map $L \times G/H \to L \to M$ induces the map $\pi_E: E \to M$, and a differential structure is introduced in E in a natural manner by using π_E (see [4]). The surjective map $(l, \xi) \mapsto \xi\cdot l$ of $L \times G/H$ onto L/H

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