

THE CONTACT OF SPACES WITH CONNECTION

IVAN KOLÁŘ

É. Cartan [1] investigated the contact of a space with projective connection with the corresponding projective space. His definition of contact of order r is based on the developments of individual curves by means of a connection, and can be easily extended to the case of two arbitrary "generalized" spaces with connection of the same type. In this paper, we first show the existence of another natural point of view from which the problem of contact may be studied based on the developments by means of successive prolongations of a connection according to Ehresmann [3]. Since the second condition is stronger, we speak of strong and weak contacts of generalized spaces with connection. The comparison of these two points of view leads us to the definition of γ -equivalence of semi-holonomic jets. To treat this problem we introduce an invariant symmetrization of some special semi-holonomic jets. Further, we remark that one can also distinguish between strong and weak deformations of order r for generalized spaces with connection. Finally, we pose a natural generalization of the original problem of É. Cartan by studying the contact of a space with Cartan connection with the corresponding homogeneous space. We treat both strong and weak contacts. Our Propositions 8 and 9 give generalizations of the results of É. Cartan [1, pp. 189, 193]. We hope that these results together with the corresponding methods illustrate clearly the fact that prolongations of a connection of first order can be applied to the solution of some natural problems in the general theory of spaces with connection. Further results in this direction can be found in [5] and [6].

We intend to carry out our investigations in a direct geometric form. That is why we introduce a connection of first order on the groupoid associated with a principal fibre bundle and not on a principle fibre bundle itself. Standard terminology and notation of the theory of jets are used throughout the paper; see, e.g., [9]. In addition, j_r^s means the canonical projection of r -jets onto s -jets, $s < r$. Our considerations are carried out in the category C^∞ .

1. Preliminaries

Let $P(B, G, \pi)$ be a principal fibre bundle, and PP^{-1} the groupoid associated with P . An element of a connection of first order on PP^{-1} at $x \in B$ can be in-

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