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THE NULLITY SPACES OF CURVATURE-LIKE TENSORS

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1. Introduction

In this paper we use the methods of the author [7] to prove the following general theorem on the nullity spaces of tensor fields which have all the formal properties of the curvature tensor. This result may be regarded as an intrinsic version of the immersion theorem proved in [7]. See § 2 for precise definitions.

Theorem 1. *Let K denote a curvature-like tensor on a Riemannian manifold M, and μ_K its index of nullity. Assume M is complete, and let G_K be the open set on which μ_K takes its minimum value m (assumed > 0). Then every leaf L of the nullity foliation of K induced on G_K is complete (and totally geodesic in M).*

This theorem generalizes results given in Maltz [6] and Clifton-Maltz [3], and our proof, besides being considerably simpler, corrects an expository error in those papers. Our result also includes recent results of Abe [1] (given here as Corollaries 1, 2 and parts of Corollaries 3 and 4), obtained by refining the proofs in [6] and [3].

In § 3 we consider some well-known examples of curvature-like tensors, obtaining the following results as corollaries of Theorem 1.

Corollary 1. *Let M be a complete Riemannian manifold, and let $K_{XY} = R_{XY} - k(X \wedge Y)$. Then the leaves of the nullity foliation of K induced on G_K are complete totally geodesic submanifolds with constant curvature k.*

Corollary 2. *Let M be a complete Kählerian manifold, and let $K_{XY} = R_{XY} - (k/4)(X \wedge Y + JX \wedge JY + 2\langle X, JY\rangle J)$. Then the leaves of the nullity foliation of K induced on G_K are totally geodesic submanifolds with constant holomorphic curvature k.*

The following result gives a second class of examples.

Theorem 2. *Let $I\colon M \to \bar{M}$ be an isometric immersion of M into another Riemannian manifold $\bar{M}$. Suppose that $P^{\perp}\bar{R}_{XY}Z = 0$ for all vector fields X, Y, Z tangent to M (X is identified with I_*X, etc., as in [7]; $P^{\perp}$ denotes projection normal to M). Then the curvature difference tensor $D_{XY} = R_{XY} - \bar{R}_{XY}$ on M is curvature-like.*

It is interesting to note that the condition $P^{\perp}\bar{R}_{XY}Z = 0$ is precisely what was needed to prove the immersion theorem of [7], so this type of immersion has particularly nice properties. Examples are provided by immersions into

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