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A COMPLEX ANALOGUE OF HARTMAN-NIRENBERG CYLINDER THEOREM

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1. Introduction

Hartman and Nirenberg [3] proved, in 1959,

Theorem (*Hartman-Nirenberg*). *Let M^n be a connected complete Riemannian hypersurface in an $(n+1)$-dimensional Euclidean space R^{n+1}. If the rank of the Gauss map is ≤ 1 everywhere, then M^n is cylindrical.*

This theorem is the first global determination of a flat hypersurface M in Euclidean space. Indeed the condition about the rank of Gauss map is equivalent to the flatness of M. Classically, we had only the local classification of flat surfaces. In this paper, we shall show the complex version of the above theorem.

Let M^n be a complex n-dimensional complete connected Kählerian hypersurface isometrically and holomorphically immersed by f into an $(n+1)$-complex space C^{n+1}.

Let $\tilde{\phi}\colon M^n \to CP^n$ be a mapping from M^n to the complex projective n-space CP^n which assigns to a point x in M^n the normal plane of $f(M^n)$ at $f(x)$ in C^{n+1}, which we can identify with a point in CP^n by the parallel displacement in C^{n+1}. We call this mapping *the Gauss map* for the complex hypersurface M^n in C^{n+1}.

Let ξ be any unit normal vector field around x, and denote by A the tensor field of type (1,1) given by

$$\tilde{\nabla}_X \xi = -A_\xi X + \hat{\nabla}_X \xi \,,$$

where $\tilde{\nabla}$ is the canonical connection of C^{n+1} and $\hat{\nabla}$ is the normal connection induced by $\tilde{\nabla}$. Then we have:

(1.1) $\tilde{\phi}_*(X) = 0$ if and only if $AX = 0$, where $\tilde{\phi}_*$ is the Jacobian of $\tilde{\phi}$;

(1.2) the rank of $\tilde{\phi}_*$ is equal to that of A;

(1.3) the Gauss map $\tilde{\phi}$ is anti-holomorphic.

For the proof of (1.1), (1.2) and (1.3), see K. Nomizu and B. Smyth [5]. Now our main theorem is stated as follows.

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