J. DIFFERENTIAL GEOMETRY
7 (1972) 371-391

G-TOTAL CURVATURE OF IMMERSED MANIFOLDS

BANG-YEN CHEN

Given an immersion $x: M \to E^m$ of a bounded manifold M of dimension n in a euclidean space E^m of dimension m, we define what we call the G-total curvature with respect to a given vector-valued function g on the normal boundle B_v as the integral over B_v of g times a power of a general mean curvature, i.e., $\int_{B_v} g(K_i)^m dV \wedge d\sigma$. We also define the G-total absolute curvatures in a similar way. The main purpose of this paper is to give the relations between different G-total curvatures or G-total absolute curvatures depending on g, i and m, first for a fixed immersion and later for different immersions. In particular, our results generalize many well-known results in differential geometry such as Gauss-Bonnet's formula, Chern-Lashof's theorems, Minkowski-Hsiung's formulas, etc.

1. Definitions

Throughout this paper, a bounded manifold means a compact manifold with or without smooth boundary. A closed manifold is a (compact) bounded manifold without boundary. Let M be a bounded manifold of dimension n, and $x: M \to E^m$ an immersion of M into a euclidean space E^m of dimension m. Suppose that E^m is oriented. By a frame $P, e_1, \cdots, e_m$ in the space E^m we mean a point $P \in E^m$ and an ordered set of mutually perpendicular unit vectors $e_1, \cdots, e_m$ with an orientation coherent with that of the space E^m. Let $F(E^m)$ be the set of all frames in the space E^m, and $F(M)$ be the set of all (orthonormal) frames in M with respect to the induced metric on M.

To avoid confusion, we shall use the following ranges of indices throughout this paper unless otherwise stated:

$$1 \leq i, j, k, \cdots \leq n; \quad n+1 \leq r, s, t, \cdots \leq m; \quad 1 \leq A, B, C, \cdots \leq m .$$

In $F(E^m)$ we introduce the 1-forms θ_A, θ_{AB} by

Received June 26, 1970, and, in revised form, December 27, 1971. This paper is a condensation of the author's Ph. D. dissertation submitted to the Graduate School of the University of Notre Dame, June 1970. The author wishes to express his hearty thanks to his thesis advisor Professor T. Nagano for his constant encouragement and valuable suggestions. He is also very thankful to Professors Y. Matsushima and T. Otsuki.