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A CLASS OF VARIATIONALLY COMPLETE REPRESENTATIONS

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Introduction

Let M be a complete Riemannian manifold on which a compact connected Lie group K acts as a group of isometries. If $M = \boldsymbol{R}^n$, then K has a fixed point, hence we lose no generality in assuming for this case that the action of K is a linear orthogonal representation.

Bott and Samelson [5] have defined the concept of variational completeness. Roughly speaking, the action of K on M is variationally complete if it produces enough Jacobi fields along geodesics to determine the multiplicities of focal points to the K-orbits. This notion remains interesting and useful for the case $M = \boldsymbol{R}^n$ (e.g., cf. [4], [5]).

In [6] we formulated the notion of a "K-transversal domain". This is a closed connected flat totally geodesic imbedded submanifold $T \subset M$ which meets all K-orbits and is orthogonal to every K-orbit at each point of intersection. We showed that the existence of a K-transversal domain implies variational completeness, and deduced strong structure theorems for the singular set, the Weyl group, and the Bott-Samelson K-cycles. For $M = \boldsymbol{R}^n$, such a T is evidently a linear subspace.

The theorems of [6], applied to the case $M = \boldsymbol{R}^n$, show that those orthogonal respresentations of K which admit a K-transversal domain bear striking resemblances to the isotropy representations associated to compact symmetric spaces (hereafter referred to as *s-representations*). Indeed, s-representations constitute the principal class of known examples. This suggests that further such analogies should be sought and exploited, the ultimate aim being a complete structure theory and classification.

We will present here three theorems which advance the above program. For this purpose we employ a linear map (due to Kostant)

$$R\colon \Lambda^2(\boldsymbol{R}^n) \to \boldsymbol{k} = \text{Lie algebra of } K\ ,$$

which will be called the curvature tensor of the representation. R is defined for an arbitrary orthogonal representation of K, and in the case of an s-representation it will actually coincide with the Riemann tensor [9]. Usually

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