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AN ABSTRACT FORM OF THE NONLINEAR CAUCHY-KOWALEWSKI THEOREM

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Introduction

Consider the initial value problem for functions $u(t, x)$:

$$(1)\qquad \partial_t^m u = f(t, x, u, \partial_x^\alpha \partial_t^j u)\ , \quad \partial_t^k u|_{t=0} = \phi_k(x)\,; \quad k = 0, \cdots, m-1.$$

Here $x \in \Omega \subset R^n$, $t \in R^1$, and u may be vector valued $u = (u^1, \cdots, u^N)$; f is a nonlinear (N-vector) function depending on t, x, u and all of its derivatives of order $\leq m$ of the form $\partial_x^\alpha \partial_t^j u$, $|\alpha| + j \leq m$, $j < m$. If f is analytic in all its arguments and if the ϕ_k are analytic, then the Cauchy-Kowalewski theorem asserts the existence of a unique analytic solution in a neighborhood of any initial point $(x_0, 0)$.

In the case of linear system of equations several people have observed independently that it is not necessary to assume analyticity in t, i.e., if f is merely continuous in t (with values as an analytic function of the other variables), there exists a unique solution $u(t, x)$ continuously differentiable in t with values in analytic functions of x—in a neighborhood of $(x_0, 0)$. This result has been put into a general, abstract, framework by T. Yamanaka [8] and again by L. V. Ovsjannikov [5] (see J. F. Treves [6] for an exposition and many applications). This result and its proof are direct extensions of the corresponding result and proof for equations with coefficients independent of t of Gelfand, Silov [2]; it is described below in Theorem A.

In [7] (see also [1]) Treves has presented a nonlinear form of the abstract Cauchy-Kowalewski theorem; it is not strong enough, however, to prove existence (and uniqueness) for (1) in the case that f is only continuous in t as an analytic function of the other variables. In this paper we present a nonlinear form of the abstract result which can be applied to this case. After completion of this work we learned that, in fact, this case had been solved by M. Nagumo [4] in 1941. Our result is stated in § 1 and proved in § 2; for completeness the application to (1) is then presented in § 3. Our proof makes use of Newton's iteration method and follows the ideas of J. Moser [3]. In § 4 we also present an implicit function theorem which is essentially just an abstract setting of a

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