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# SOME METRIC PROPERTIES OF ARITHMETIC QUOTIENTS OF SYMMETRIC SPACES AND AN EXTENSION THEOREM

ARMAND BOREL

This paper has two main objectives. One is to prove:

**Theorem A.** *Let $D$ be the open unit disc $|z| < 1$ in $\boldsymbol{C}$, and $D^* = D - \{0\}$. Let $X$ be a bounded symmetric domain, and $\Gamma$ an arithmetically defined torsion-free group of automorphisms of $X$. Let $V^*$ be the complex analytic compactification of $V = X/\Gamma$ constructed in* [3], a *and* b *positive integers, and $f\colon D^{*a} \times D^b \to V$ a holomorphic map. Then $f$ extends to a holomorphic map of $D^{a+b}$ into $V^*$.*

In fact, a slightly more general result will be obtained (see Thm. 3.7). Together with some known facts, this implies that if $S$ is an algebraic variety, $h\colon S \to V$ is a holomorphic map, and $V$ is endowed with its natural structure of quasi-projective variety defined in [3, Thm. 3,10], then $h$ is a morphism of algebraic varieties.

The proof of Theorem A makes use of an extension theorem of M. H. Kwack [12], or rather of a slight variant of it [9], and the main point is to check that its assumptions are satisfied in our case. Let $d_0$ be the Kobayashi invariant pseudo-distance [10] on $X$; since $X$ is a bounded symmetric domain, it is a distance (cf. § 3.3). Let $d_0'$ be the associated distance on $V$ defined by

$$d_0'(\pi(x), \pi(y)) = \inf_{\gamma \in \Gamma} d_0(x, y\cdot\gamma) , \qquad (x, y \in X) , \tag{1}$$

where $\pi\colon X \to V$ is the canonical projection. In view of some distance decreasing properties of $f$, we have essentially to prove the following result (where $\Gamma$ may have torsion):

**Theorem B.** *Let $p, q \in V^* - V$, and let $p_n, q_n$ $(n = 1, 2, \cdots)$ be sequences of points in $V$ converging to $p$ and $q$ respectively. If $d_0'(p_n, q_n) \to 0$, then $p = q$.*

Theorem B will be derived in § 3.5 from properties of Siegel sets and arithmetic groups, whose discussion is the other purpose of this paper. Since they have some independent interest, they will be proved in greater generality and in a stronger form than is needed in § 3.5. Let then $\Gamma$ be an arithmetic subgroup of a connected semi-simple $\boldsymbol{Q}$-group $\mathscr{G}$, $X$ the symmetric space of maximal compact subgroups of the group $G$ of real points of $\mathscr{G}$, and $d_X$ the

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