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BOUNDARY BEHAVIOR OF $\bar{\partial}$ ON WEAKLY PSEUDO-CONVEX MANIFOLDS OF DIMENSION TWO

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1. Introduction

The problem of finding holomorphic functions in a domain which cannot be extended across the boundary, usually known as the Levi problem, seems to be intimately connected with various regularity properties of the operator $\bar{\partial}$. We will deal here with a complex manifold M with a smooth boundary, denoted by bM. Consider the following local version of the Levi problem: given $P \in bM$ find a holomorphic function in a neighborhood of P whose restriction to $\overline{M}$ vanishes exactly at P. A classical result states that whenever the Levi form is positive definite the problem has a solution, but if the Levi form has a negative eigenvalue or is identically zero in a neighborhood of P then the problem does not have a solution. This behaviour of the Levi form also controls the local (or more precisely the pseudo-local) regularity of the inhomogenous Cauchy-Riemann operator $\bar{\partial}$. It is natural to ask: what happens when the Levi form is positive semi-definite, vanishes at P, but not identically in a neighborhood. Here we establish some conditions for the solution of these problems.

We shall investigate the regularity properties of $\bar{\partial}$ by means of the $\bar{\partial}$-Neumann problem. On this occasion we do not wish to recall the history of this problem; we refer to [1] for a selfcontained treatment of the $\bar{\partial}$-Neumann problem as well as an historical discussion. However, since this paper is dedicated to Professor Spencer's sixtieth birthday, it is appropriate to point out that the $\bar{\partial}$-Neumann problem was first formulated by D. C. Spencer and that he pioneered several of its applications and generalizations to overdetermined systems. We shall impose some conditions on M and establish certain "subelliptic estimates" for the $\bar{\partial}$-Neumann problem. Among the consequences of such estimates are the following:

(i) Existence, regularity and pseudo-localness of a solution to the inhomogeneous Cauchy-Riemann equations. That is, whenever the above mentioned estimates hold, there is a unique solution of the equation $\bar{\partial}u = \alpha$ (where α is a (0, 1)-form satisfying the necessary compatibility condition), such that u is

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