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FUNCTION THEORY OF FINITE ORDER ON ALGEBRAIC VARIETIES. I (A)

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1. Introduction

The purpose of this paper is to discuss the theory of analytic functions of finite order on algebraic varieties. The proofs of our results are only sketched as the complete arguments will appear in a more general setting at a later time.

There are two reasons for making this study. The first is that the classical theory of meromorphic functions of finite order [19] and the extensions of this theory to functions on $\boldsymbol{C}^n$, [18], [24], form a very pretty subject, and one which furnishes us with the most important examples of entire transcendental functions for use in analysis and number theory. As the natural domains of existence for functions of finite order are the algebraic varieties, it seems worthwhile to develop the theory in this setting. The second, and more important, reason is that on an affine algebraic variety A the functions of finite order give the smallest class of functions which might allow one to realize the topological Grothendieck ring $K_{\text{top}}(A)$. Or, to put matters another way, since A is a Stein manifold, Grauerts' proof of the Oka principle [6] gives the isomorphism

$$K_{\text{top}}(A) \cong K_{\text{hol}}(A)$$

between the topological and analytic K-theories on A. What the examples and partial results the author has seem to indicate is the refined isomorphism

$$K_{\text{top}}(A) \cong K_{\text{f.o.}}(A) \tag{1.1}$$

between the topological and finite order K-theories on A. We are able to establish the isomorphism (1.1) in certain special cases, and in general are able to reduce the problem to semi-local questions in several complex variables. If (1.1) were established, then we could be able to measure the obstructions to making an analytic cycle $\Gamma \in H^{2q}(A, \boldsymbol{Q})$ progressively more algebraic. Some examples suggest (roughly) that the μ^{th} obstruction should be the projection of Γ in $\sum_{|r-s|>\mu} H^{r,s}(A, \boldsymbol{C})$ (here we are using Deligne's mixed Hodge structure [5] on

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