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THE CONJECTURES ON CONFORMAL TRANSFORMATIONS OF RIEMANNIAN MANIFOLDS

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Introduction

Let (M, g) be a Riemannian n-manifold with Riemannian metric g. *Throughout this paper manifolds under consideration are always assumed to be connected and smooth.*

For a smooth function ρ on M, a Riemannian metric $e^{2\rho}g$ is said to be *conformally related*, or *conformal*, to g. Let h be a smooth map of (M, g) into another Riemannian manifold (M', g'). If the Riemannian metric h^*g' induced on M by h is conformal to g, then h is called a *conformal map* of (M, g) into (M', g'). It is well-known that h is conformal if and only if it preserves the angle between any two tangent vectors. h remains conformal under any conformal changes of Riemannian metrics on M and M' as well. If h is a conformal diffeomorphism of (M, g) onto (M', g'), then it is called briefly a *conformorphism* of (M, g) onto (M', g'), and (M, g) is said to be *conformorphic* to (M', g') Via h. If furthermore $(M, g) = (M', g')$, then h is called a *conformal transformation* or a *conformorphism* of (M, g).

It is known that the group $C(M, g)$ of all conformorphisms of (M, g) is a Lie group with respect to the compact-open topology. Let $C_0(M, g)$ denote the connected component of the identity of $C(M, g)$. If g and $\bar{g}$ are conformal to each other, then $C(M, g) = C(M, \bar{g})$. The group $I(M, g)$ of all isometries of (M, g) is a closed subgroup of $C(M, g)$. A subgroup G of $C(M, g)$ is said to be *essential* if G is not contained in $I(M, e^{2\rho}g)$ for any smooth function ρ on M, and is said to be *inessential* otherwise.

In this paper, unless otherwise stated, *we always assume* $\dim M > 2$, although some of our propositions are valid even for $\dim M = 2$.

There have been two conjectures:

Conjecture I. *Let (M, g) be a compact Riemannian n-manifold. Then $C_0(M, g)$ is essential if and only if (M, g) is conformorphic to a Euclidean n-sphere S^n.*

Conjecture II. *Let (M, g) be a compact Riemannian n-manifold with constant scalar curvature k. Then $C_0(M, g)$ is essential if and only if k is positive and (M, g) is isometric to a Euclidean n-sphere $S^n(k)$ of radius* $1/\sqrt{k}$.

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