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ORIENTATION OF DIFFERENTIABLE MANIFOLDS

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0. Introduction

We shall study compact, connected C^∞-manifolds M_n provided with a Riemannian metric. Homologically characterized, an "orientable" manifold M_n is a manifold whose n-th Betti number is 1, or equivalently a manifold whose n-th connectivity over the field $\mathbf{Q}$ of rational numbers is 1. If the manifold M_n is triangulated, another and equivalent characterization is that the simplicial cells of M_n can be *coherently oriented*, in the classical sense.

In [6] the authors concern themselves with a systematic development of singular homology on M_n without making use of any triangulation of M_n. Triangulations are avoided for two reasons. In a study of *ND* (abbreviating non-degenerate) functions on M_n it is found that a global triangulation is neither needed nor relevant. A deeper reason is that the methods of the critical point theory, if developed without any use of global triangulations of M_n, are extendable to compact, connected topological manifolds admitting a topologically *ND* function. See [4], [8] and [7]. For a definition of topologically *ND* functions see [1].

Objective. We shall give a *geometic* definition of the orientability of M_n. This definition has many consequences in the study of *ND* function on M_n. In particular one can show, without making use of any global triangulation of M_n, that M_n is geometrically orientable in our sense if and only if $\beta_n(M_n)=1$. It is believed, moreover, that the theory here developed for differentiable manifolds has an extension to topological manifolds admitting a topologically *ND* function. The theorems on "critical shells", introduced in § 7 when $n > 2$ and f is "biorderd" (§ 4), are believed to be fundamental both in the orientable and nonorintable case.

1. Inverting sequences of presentations

Definition 1.0. $\pm$*Compatibility*[1]. Two overlapping presentations Q and F in $\mathscr{D}M_n$ (see [6, § 13]) will be said to be $\overset{+}{\text{Com}}$ ($\overset{-}{\text{Com}}$) if the transition diff λ associated with Q and F (see [6, § 5]) has a positive (negative) Jacobian at each point of the euclidean domain of definition of λ. The intersection of the ranges

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[1] Compatible will be abbreviated by Com.