J. DIFFERENTIAL GEOMETRY
5 (1971) 523-542

SINGULAR MANIFOLDS

MICHAEL MENN

1. Introduction

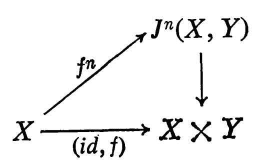
If $\phi: X \to Y$ is a map of topological spaces and $x \in X$, then ϕ_x will denote the germ of ϕ at x. Let $\mathfrak{F}(p, q) = \{\phi: \boldsymbol{R}^p \to \boldsymbol{R}^q \mid \phi \text{ is } \mathscr{C}^\infty \text{ and } \phi(0) = 0\}$ and let $J(p, q) = \{\phi_0 \mid \phi \in \mathfrak{F}(p, q)\}$. If $\phi \in \mathfrak{F}(p, q)$ or $\phi \in J(p, q)$, then $[\phi]^n$ will denote the set of germs at the origin of elements of $\mathfrak{F}(p, q)$, which agree with ϕ up to and including order n at the origin. $[\phi]^n$ will occasionally be abbreviated to ϕ. Let $J^n(p, q) = \{[\phi]^n \mid \phi \in J(p, q)\}$.

Whenever m is an integer, $\mathscr{L}_m$ will denote the set of invertible germs in $J(m, m)$. $\mathscr{L}_m$ is a group. Furthermore, there is a group action of $\mathscr{L}_p \times \mathscr{L}_q$ on $J^n(p, q): (\alpha, \beta)([\phi]^n) = [\beta\phi\alpha^{-1}]^n$. Suppose $\phi: U \to \boldsymbol{R}^q$ is $\mathscr{C}^\infty$ where U is an open subset of $\boldsymbol{R}^p$. Define $t_\phi: U \to J(p, q)$ by $t_\phi(x)$ is the germ at the origin of $y \to \phi(x + y) - \phi(x)$. In the following all manifolds are $\mathscr{C}^\infty$ and paracompact, and all maps are $\mathscr{C}^\infty$.

Let $\tilde{\mathscr{L}}_m$ be a subgroup of $\mathscr{L}_m$. Suppose M is an m-dimensional manifold and $\mathscr{A}$ is an atlas of coordinate functions for M. The pair $(M, \mathscr{A})$ will be called a manifold of type $\tilde{\mathscr{L}}_m$ if for all $x \in M$ and coordinate functions $\alpha_1, \alpha_2 \in \mathscr{A}$ whose domains contain x, $t_{\alpha_2\alpha_1^{-1}}(\alpha_1(x)) \in \tilde{\mathscr{L}}_m$. The atlas $\mathscr{A}$ will be suppressed from the notation.

Let X be a p-manifold and Y a q-manifold. $J^n(X, Y)$ will be the bundle with base $X \times Y$, fiber $J^n(p, q)$, and group $\mathscr{L}_p \times \mathscr{L}_q$. Let $\tilde{\mathscr{L}}_p$ be a subgroup of $\mathscr{L}_p$ and $\tilde{\mathscr{L}}_q$ a subgroup of $\mathscr{L}_q$. Suppose X is a manifold of type $\tilde{\mathscr{L}}_p$ and Y is a manifold of type $\tilde{\mathscr{L}}_q$. Then the group of $J^n(X, Y)$ is reducible to $\tilde{\mathscr{L}}_p \times \tilde{\mathscr{L}}_q$. $J^n(X, Y)$ may be looked at as the set of equivalence classes of germs of maps of X into Y where two germs are equivalent if they agree up to order n.

If $f: X \to Y$ and $x \in X$, then $f^n(x)$ will denote the equivalence class containing the germ of f at x. Thus a map $f: X \to Y$ induces a commutative triangle:



Communicated by J. J. Kohn, October 12, 1968, and, in revised form, January 26, 1969. This work is part of the author's doctoral thesis written at Brandeis University under the direction of H. I. Levine; several of his suggestions and simplifications were adopted.