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RIEMANNIAN MANIFOLDS ADMITTING A CERTAIN CONFORMAL TRANSFORMATION GROUP

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1. Introduction

Several authors have studied compact Riemannian manifolds admitting a conformal non-Killing vector field. The main results are as follows.

Let M be a connected n-dimensional Riemannian manifold admitting a conformal non-Killing vector field.

(1) *If M is a complete Einstein space of dimension $n \geq 3$, then M is isometric to a sphere* (Nagano-Yano [8]).

(2) *If M is a complete Riemannian manifold of dimension $n \geq 3$ with parallel Ricci tensor, then M is isometric to a sphere* (Nagano [5]).

(3) *If M is compact and homogeneous, then M is isometric to a sphere provided $n > 3$* (Goldberg-Kobayashi [2]).

(4) *M can not be a compact Riemannian manifold with constant nonpositive scalar curvature* (Yano [7], Lichnerowicz [4]).

Recently S. Tanno and W. C. Weber [6] investigated compact connected Riemannian manifolds which have constant scalar curvature and admit a closed conformal vector field with certain conditions. The purpose of this paper is to prove the following theorems.

Theorem I. *If a compact connected Riemannian manifold M admits a closed conformal non-Killing vector field, then M is diffeomorphic to a generalized twisted torus or a sphere.*

Theorem 2. *If a compact Riemannian manifold M with finite fundamental group admits a closed conformal non-Killing vector field, then M is diffeomorphic to a sphere.*

Theorem 3. *If a compact connected Riemannian manifold M admits a closed conformal non-Killing vector field which vanishes at some point of M, then M is diffeomorphic to a sphere.*

Theorem 2 is an immediate consequence of Theorem 1, and Theorem 3 follows from the proof of Theorem 1.

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