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THE ABSOLUTE AND RELATIVE BETTI NUMBERS OF A MANIFOLD WITH BOUNDARY

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1. Consider a compact manifold M with a boundary B, so that M is the closure of an open submanifold of an n-dimensional orientable Riemannian manifold V, and B is a compact orientable $(n-1)$-dimensional manifold. Let $H_p(M, \boldsymbol{R})$ and $H_{n-p}\{(M, B), \boldsymbol{R}\}$ be respectively the pth Betti group of M and the pth Betti group of M(mod. B). Then by Lefschetz duality theorem the pth Betti group $H_p(M, \boldsymbol{R})$ and the $(n-p)$th Betti group $H_{n-p}\{(M, B), \boldsymbol{R}\}$ are dual, so that the absolute Betti number A_p and the relative Betti number R_{n-p} of the manifold M are equal. For a k-pinched manifold M, the numbers A_p and R_q for $p = q = 2$ are zero, when the number k is greater than a number λ and the second fundamental form on B satisfies some conditions. We can improve the number λ, when the dimension of the manifold M is 5. These results are a generalization of those given in [8].

2. If α, β are two tensors of the manifold M of order p, then the local inner product of the two tensors α, β is defined by

$$(\alpha, \beta) = \frac{1}{p!}\alpha^{i_1\cdots i_p}\beta_{i_1\cdots i_p} = \frac{1}{p!}\alpha_{i_1\cdots i_p}\beta^{i_1\cdots i_p} ,$$

and the local norm of the tensor α is defined by

$$|\alpha|^2 = \frac{1}{p!}\alpha^{i_1\cdots i_p}\alpha_{i_1\cdots i_p} .$$

If η is the volume element of the manifold M, then the global inner product of the two tensors α, β and the global norm of the tensor α are defined, respectively,

$$\langle\alpha, \beta\rangle = \int_M (\alpha, \beta)\eta , \qquad \|\alpha\|^2 = \int_M |\alpha|^2\eta .$$

If α is a p-form, then we have [6, p. 187]

$$\langle \Delta\alpha, \alpha\rangle = \|d\alpha\|^2 + \|\delta\alpha\|^2 , \tag{2.1}$$

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