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CURVATURE AND THE EIGENFORMS OF THE LAPLACE OPERATOR

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1. Introduction

Let X be a d-dimensional compact oriented Riemannian manifold of class C^∞ without boundary, $\wedge^p$ the space of smooth exterior p-forms, $d\colon \wedge^p \to \wedge^{p+1}$ the operator of exterior differentiation, $d^*\colon \wedge^{p+1} \to \wedge^p$ the adjoint of d with respect to the Riemannian metric, and $\Delta = -(dd^* + d^*d)$ the Laplace operator acting on exterior p-forms for $0 \le p \le d$. It is known that $\Delta\colon \wedge^p \to \wedge^p$ has an infinite sequence

$$0 \geq \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \cdots \downarrow -\infty$$

of eigenvalues, each of which is repeated as many times as its multiplicity indicates, and the corresponding sequence $\{\varphi_n\}$ of eigenforms forms a complete orthonormal set in the space $\wedge^p$ with Riemannian inner product. The sum

$$\sum_{n\geq 0} \exp(\lambda_n t)\varphi_n(x) \otimes \varphi_n(y)$$

converges uniformly on compact subsets of $(0, \infty) \times X$ to the fundamental solution $e^p(t, x, y)$ of the operator $\partial/\partial t - \Delta$ acting on p-forms, and the trace $Z^p = \sum_{n\geq 0} \exp(\lambda_n t)$ can be expressed as the integral over the manifold of the pole $\operatorname{Tr} e^p = \sum_{n\geq 0} \exp(\lambda_n t)\langle\varphi_n, \varphi_n\rangle$, $\langle\varphi_n, \varphi_n\rangle$ being the Riemannian inner product of p-forms at a point of X, that is,

$$Z^p = \int_X \operatorname{Tr} e^p .$$

Let Z be the alternating sum of Z^p, that is,

$$Z = \sum_{p=0}^{d} (-1)^p Z^p ,$$

and $\operatorname{Tr} e = \sum_{p=0}^{d} (-1)^p \operatorname{Tr} e^p$. It is proved in [2] that

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