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SCALAR CURVATURE OF COMPLEX SUBMANIFOLDS OF A COMPLEX PROJECTIVE SPACE

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1. Statement of results

Let $P_{n+p}(\mathbf{C})$ be a complex projective space of complex dimension $n + p$ with the Fubini-Study metric of constant holomorphic sectional curvature 1, and X be an n-dimensional compact complex submanifold of $P_{n+p}(\mathbf{C})$ with the induced Kaehler structure. Then X is algebraic by a well known theorem of Chow. Throughout this paper, we assume that X is a complete intersection of p hypersurfaces in general position in $P_{n+p}(\mathbf{C})$, i.e., that there exist p hypersurfaces $X_1, \cdots, X_p$ of degree $a_1, \cdots, a_p$ in $P_{n+p}(\mathbf{C})$ such that $X = X_1 \cap \cdots \cap X_p$. As a matter of course, every compact complex hypersurface in $P_{n+p}(\mathbf{C})$ is under consideration. The purpose of the present paper is to prove the following results:

Theorem. *Let X be a complete intersection of p hypersurfaces of degree $a_1, \cdots, a_p$ in general position in $P_{n+p}(\mathbf{C})$, and ρ be the scalar curvature of X. Then*

$$\int_X \rho * 1 = n\{n + p + 1 - (a_1 + \cdots + a_p)\} \int_X * 1 ,$$

where $*\,1$ denotes the volume element of X.

This theorem implies that the average of the scalar curvature depends only on the degree of X, while the scalar curvature itself on the equations defining X.

Corollary 1. *If $\rho > n^2$ everywhere on X, then $X = P_n(\mathbf{C})$.*

Corollary 2. *Let X be a hypersurface of $P_{n+1}(\mathbf{C})$. If $n(n - \nu + 1) < \rho \leq n(n - \nu + 2)$ everywhere on X, then X is an algebraic manifold of degree ν.*

Let S be the square of the length of the second fundamental form of the imbedding so that $S = n(n + 1) - \rho$. The following corollaries are equivalent to Corollary 1 and Corollary 2 respectively.

Corollary 1′. *If $S < n$ everywhere on X, then $X = P_n(\mathbf{C})$.*

Corollary 2′. *Let X be a hypersurface of $P_{n+1}(\mathbf{C})$. If $n(\nu - 1) \leq S < n\nu$ everywhere on X, then X is an algebraic manifold of degree ν.*

In a previous paper [3], we have proved that *if $S < (n + 2)/(4 - 1/p)$ everywhere on X, then $X = P_n(\mathbf{C})$*. Corollary 1′ is an improvement of this result and is best possible for the following reason: Let $Q_n(\mathbf{C}) = \{(z_0, \cdots, z_{n+1}) \in P_{n+1}(\mathbf{C}) \mid \sum z_i^2 = 0\}$, where $z_0, \cdots, z_{n+1}$ be the homogeneous coordinates

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