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ISOMETRIC IMMERSIONS OF RIEMANNIAN PRODUCTS

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Introduction

For each integer i, $1 \leq i \leq p$, let M_i be a compact connected riemannian manifold of dimension $n_i \geq 2$, and M the riemannian product $M_1 \times M_2 \times \cdots \times M_p$. In this paper we will prove that any codimension p isometric immersion of M in euclidean space is a product of hypersurface immersions. This means that if $f\colon M \to E^N$ is an isometric immersion into euclidean space E^N of dimension $N = \left(\sum_{i=1}^{p} n_i\right) + p$, then there exist isometric immersions $f_i\colon M_i \to E^{n_i+1}$ $(1 \leq i \leq p)$ and a decomposition of E^N into a riemannian product

$$E^N = E^{n_1+1} \times \cdots \times E^{n_p+1}$$

so that $f(m_1, m_2, \cdots, m_p) = (f_1(m_1), f_2(m_2), \cdots, f_p(m_p))$ when $m_i \in M_i$ for $1 \leq i \leq p$. This generalizes a result of S. B. Alexander [1] which dealt with codimension two isometric immersions.

As an application, we mention that if S^2 is the two-dimensional sphere of constant curvature one, then it follows from Liebmann's theorem that the riemannian product $S^2 \times S^2 \times \cdots \times S^2$ (p times) is globally rigid in $3p$-dimensional euclidean space E^{3p}; it is clearly not locally rigid. Very few global rigidity theorems in codimensions higher than one are known, and this example is perhaps the simplest.

Unless otherwise stated all riemannian manifolds are C^∞ and connected, and we use [5] as a reference for the basic theorems of riemannian geometry. The author sincerely thanks Professors M. P. do Carmo and S. Kobayashi for their encouragement, and the referee for several valuable suggestions. The main results in this paper were included in the author's thesis written under the direction of Professor Kobayashi at the University of California, Berkeley.

1. Statement of results

If M is a riemannian manifold, let $F(M)$ denote the interior of the set of points of M at which all sectional curvatures vanish. Our first result is proven by local methods:

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